On Explicit Partial Estimator Design for Nonlinear Systems with Parametric Uncertainties

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Problem statement

Class of Systems

$$\dot{x} = f(x, u, p) \quad | \quad y = h(x, u, p)$$

$\rightarrow$ f, h known.

$\rightarrow$ p, uncertain | known statistics $\mathcal{P}$.

$\rightarrow$ y measurement vector **including u**

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Partial observation target

$$z = T(x, u, p)$$

$\to$ T known.

$\to$ Example: Control law: $z = k(x, p)$

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Extend the state $\rightarrow$ Ext-MHE $\rightarrow \hat{x}, \hat{p} \rightarrow \hat{z}$

CPU-expensive | full estimation | non observability

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This contribution

Design $\mathcal{O}$ such that:

$$\boxed{\hat{z}_k \approx \mathcal{O}(\mathbf{y}_k^{(-)})}$$

$$\text{where} \quad \mathbf{y}_k^{(-)} := \begin{bmatrix} y_k \\ y_{k-m} \\ \vdots \\ y_{k-Nm} \end{bmatrix} \in \left[\mathbb{R}^{n_y}\right]^{N+1}$$

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This contribution

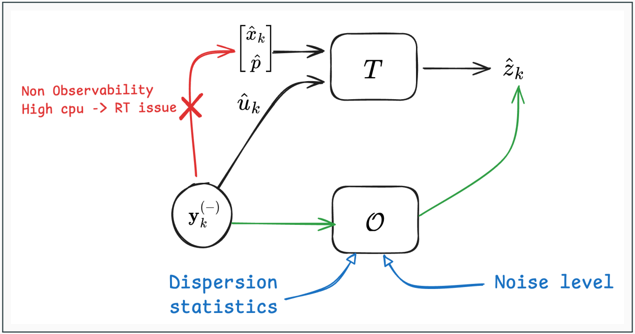
Design $\mathcal{O}$ such that:

$$\boxed{\hat{z}_k \approx \mathcal{O}(\mathbf{y}_k^{(-)})}$$

$\mathcal{O}$ must take into account:
1) The statistics $\mathcal{P}$,
2) The noise level

$$\text{where} \quad \mathbf{y}_k^{(-)} := \begin{bmatrix} y_k \\ y_{k-m} \\ \vdots \\ y_{k-Nm} \end{bmatrix} \in \left[\mathbb{R}^{n_y}\right]^{N+1}$$

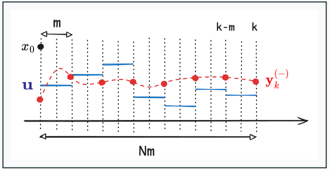
The rationale



The past measurement might retrieve the necessary information to reconstruct $z = T(x, u, p) \in \mathbb{R}$ but not necessarily $(\hat{x}, \hat{p}) \in \mathbb{R}^{n_x+n_p}$.

- ✓ Choose noise|uncertainty levels (ν, δ_p)
- ✓ Simulate n_{sc} scenarios (p sampled using $\mathcal{P}$)

$$s := (x_0, \mathbf{u}, p) \in \mathbb{X} \times \mathbb{U}^N \times \mathbb{P}(\delta_p)$$



- ✓ Collect the working data:

$$\mathcal{D}(\nu, \delta_p) := \Big\{ \big(\mathbf{y}_k^{(-)}(s) + \nu\epsilon, z_k(s)\big) \Big\}_{s,k}$$

- ✓ Fit a model $\mathcal{O}(\nu, \delta_p)$ using $\mathcal{D}(\nu, \delta_p)$.

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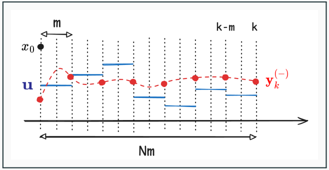

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Compare the performance on *unseen* new scenarios between:

- `Random Forest`
- `XG-Boost`
- `K-Nearest-Neighbors`
- `Deep Neural Network`
- **Plars**: Sparse polynomial (this contribution).

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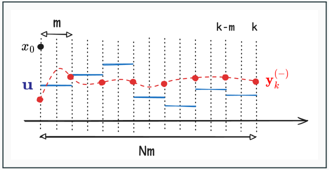

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4.1 Electronic throttle controlled system

The first example concerns the automotive Electronic Throttle Control (**ETC**) system given by (Conatser et al., 2004):

$$\dot{x}_1 = x_2 \tag{10a}$$

$$\dot{x}_2 = \frac{1}{N_m^2 J_m + J_g}\Big[\phi(x,p) + N_m K_t x_3\Big] \tag{10b}$$

$$\dot{x}_3 = \frac{1}{L_a}\Big[-N_m K_b x_2 - R_a x_3 + u\Big] \tag{10c}$$

where the state vector is given by $x := (\theta, \dot{\theta}, e_a)$ with θ standing for the air admission angle and e_a refers to the electromotor torque induced by the current $u = i_a$ (control input). The vector $p = (N_m, J_m, J_g, \dots)$ gathers all the parameters involved in (10) (see Table 1 for the values). The nonlinear map $\phi(x,p)$ appearing in (10b) is given by:

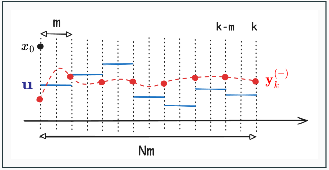
$$\phi(x,p) := -K_{sp}(x_1 - \pi/2) - (N_m^2 b_m + b_t)x_2 - \\ - 2P_{\text{atm}}(\pi - x_1)R_p^2 R_{af}\cos^2(x_1), \tag{11}$$

Building the map $\mathcal{O} : \mathrm{y}_k^{(-)} \to \hat{z}_k$ and evaluate performance

✓ Choose noise|uncertainty levels (ν, δ_p)

✓ Simulate n_{sc} scenarios (p sampled using $\mathcal{P}$)

$$s := (x_0, \mathbf{u}, p) \in \mathbb{X} \times \mathbb{U}^N \times \mathbb{P}(\delta_p)$$



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4.2 The Lorentz oscillator

This is a widely known three dimensional nonlinear system given by:

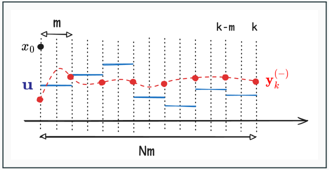
$$\dot{x}_1 = p_1(x_2 - x_1) \tag{14a}$$
$$\dot{x}_2 = x_1(p_2 - x_3) - x_2 \tag{14b}$$
$$\dot{x}_3 = x_1 x_2 - p_3 x_3 \tag{14c}$$

where the nominal parameter $p = (10, 28, 3.34)$ is used hereafter. Only the first state $y = x_1$ is supposed to be measured leaving two candidate observation targets $z_1 = x_2$ and $z_2 = x_3$. A sampling period of $\tau = 10^{-2}$ is used.

✓ Choose noise|uncertainty levels (ν, δ_p)

✓ Simulate n_{sc} scenarios (p sampled using $\mathcal{P}$)

$$s := (x_0, \mathbf{u}, p) \in \mathbb{X} \times \mathbb{U}^N \times \mathbb{P}(\delta_p)$$



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ETC

$\mathbf{y}_k^{(-)} \in \mathbb{R}^{30} \mid y = x_1 \mid z \in \{x_2, x_3\}$

6800 samples $\leftarrow$ training

136000 samples $\leftarrow$ test

✓ Choose noise|uncertainty levels (ν, δ_p)

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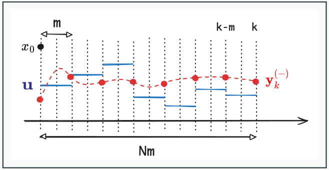

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6800 samples ← training

136000 samples ← test

Lorentz

$\mathbf{y}_k^{(-)} \in \mathbb{R}^{5} \mid y = x_1 \mid z \in \{x_2, x_3\}$

2200 samples ← training

44000 samples ← test

✓ Choose noise|uncertainty levels (ν, δ_p)

✓ Simulate n_{sc} scenarios (p sampled using $\mathcal{P}$)

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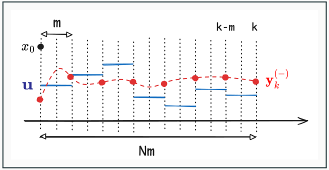

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Parameter	ETC	Lorentz
n	100	250
σ_p	$\in \{0, 0.05, 0.1\}$	$\in \{0, 0.05, 0.1\}$
x_{min}	[-0.5, -0.5, -0.5]	[-1,-1,-1]
x_{max}	[0.5, 0.5, 0.5]	[1,1,1]
t_f	3.0	4.0
N	15	5
m	2	10

Table 2. Parameters used in data generation for each pair of (z, σ_p) defining the observation target and the level of parameter dispersion.

✓ Choose noise|uncertainty levels (ν, δ_p)

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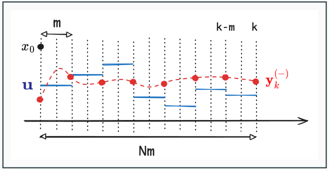

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Unfair comparison 🙂

For all competing solutions (except Plars), sets of hyper-parameters settings have been spanned and the ones leading to the best performance **(on test dataset)** have been used in the displayed performance results.

Presentation of the performance results

Results for ETC with $\mathbf{z} = \mathbf{x_2}$, $\sigma_{\mathbf{p}} = \mathbf{0.}$ and noise=0.0.

q	plars	RF	XGB	KNN	DNN
50%					
80%					
95%					
99%					

$$100 \times \left[\frac{p_q^{\texttt{algo}} - p_q^{\texttt{plars}}}{p^{\texttt{plars}} + \epsilon}\right] \quad ; \quad \epsilon = 0.001$$

$$p_q^{\texttt{plars}} := \frac{\texttt{percentile}(|z - \hat{z}|, q)}{\texttt{median}(|z|)}$$

Results

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q	plars	RF	XGB	KNN	DNN
50%	0.0	17000.0%	10600.0%	17200.0%	93400.0%
80%	0.0	23900.0%	13450.0%	24850.0%	131950.0%
95%	0.0	20140.0%	11160.0%	22200.0%	103140.0%
99%	0.01	22278.0%	11189.0%	23300.0%	83167.0%

Results for `ETC` **with $\mathbf{z}=\mathbf{x_2}$, $\sigma_\mathbf{p}=\mathbf{0.0}$ and** `noise`**=0.025.**

q	plars	RF	XGB	KNN	DNN
50%	0.04	383.0%	214.0%	383.0%	2350.0%
80%	0.1	409.0%	197.0%	423.0%	2705.0%
95%	0.24	329.0%	146.0%	363.0%	2197.0%
99%	0.51	298.0%	109.0%	318.0%	1376.0%

Results for `ETC` **with $\mathbf{z}=\mathbf{x_2}$, $\sigma_\mathbf{p}=\mathbf{0.0}$ and** `noise`**=0.05.**

q	plars	RF	XGB	KNN	DNN
50%	0.06	203.0%	108.0%	197.0%	1541.0%
80%	0.16	220.0%	99.0%	225.0%	1648.0%
95%	0.38	179.0%	70.0%	199.0%	1381.0%
99%	0.78	169.0%	46.0%	171.0%	943.0%

Fig. 1. Partial estimation results (on unseen test data) for the observation target $z=x_2$ of `ETC` system with no parametric dispersion and different levels of noise.

Results

Results for ETC with $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = 0.$ and noise=0.0.

q	plars	RF	XGB	KNN	DNN
50%					
80%					
95%					
99%					

$$100 \times \left[\frac{p_q^{\texttt{algo}} - p_q^{\texttt{plars}}}{p^{\texttt{plars}} + \epsilon}\right] \quad ; \quad \epsilon = 0.001$$

$$p_q^{\texttt{plars}} := \frac{\texttt{percentile}(|z-\hat{z}|, q)}{\texttt{median}(|z|)}$$

Results for ETC with $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = 0.05$ and noise=0.025.

q	plars	RF	XGB	KNN	DNN
50%	0.06	219.0%	125.0%	246.0%	1539.0%
80%	0.16	202.0%	101.0%	226.0%	1525.0%
95%	0.35	174.0%	80.0%	206.0%	1299.0%
99%	0.59	182.0%	82.0%	215.0%	1081.0%

Results for ETC with $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = 0.1$ and noise=0.025.

q	plars	RF	XGB	KNN	DNN
50%	0.1	145.0%	74.0%	164.0%	829.0%
80%	0.28	127.0%	56.0%	144.0%	859.0%
95%	0.58	122.0%	56.0%	139.0%	781.0%
99%	0.93	131.0%	67.0%	143.0%	655.0%

Results for ETC with $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = 0.05$ and noise=0.05.

q	plars	RF	XGB	KNN	DNN
50%	0.08	155.0%	88.0%	173.0%	1140.0%
80%	0.2	141.0%	66.0%	160.0%	1144.0%
95%	0.44	118.0%	50.0%	143.0%	987.0%
99%	0.78	114.0%	47.0%	141.0%	775.0%

Results for ETC with $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = 0.1$ and noise=0.05.

q	plars	RF	XGB	KNN	DNN
50%	0.12	118.0%	61.0%	125.0%	775.0%
80%	0.31	105.0%	47.0%	116.0%	737.0%
95%	0.65	98.0%	47.0%	112.0%	653.0%
99%	1.06	102.0%	50.0%	117.0%	579.0%

Fig. 2. Partial estimation results (on unseen test data) for the observation target $z = x_2$ of ETC system with different levels of parametric dispersion and measurement noise.

Results

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80%					
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$$p_q^{\texttt{plars}} := \frac{\texttt{percentile}(|z - \hat{z}|, q)}{\texttt{median}(|z|)}$$

Results for `lorentz` **with $\mathbf{z} = \mathbf{x_2}$, $\sigma_\mathbf{p} = \mathbf{0.05}$ and** `noise`**=0.025.**

q	plars	RF	XGB	KNN	DNN
50%	0.03	59.0%	28.0%	52.0%	3383.0%
80%	0.06	60.0%	34.0%	53.0%	1950.0%
95%	0.1	63.0%	71.0%	63.0%	1190.0%
99%	0.18	121.0%	145.0%	75.0%	1390.0%

Results for `lorentz` **with $\mathbf{z} = \mathbf{x_2}$, $\sigma_\mathbf{p} = \mathbf{0.1}$ and** `noise`**=0.025.**

q	plars	RF	XGB	KNN	DNN
50%	0.03	46.0%	31.0%	62.0%	3581.0%
80%	0.05	49.0%	38.0%	64.0%	2340.0%
95%	0.1	59.0%	61.0%	73.0%	1458.0%
99%	0.18	120.0%	130.0%	104.0%	1370.0%

Results for `lorentz` **with $\mathbf{z} = \mathbf{x_2}$, $\sigma_\mathbf{p} = \mathbf{0.05}$ and** `noise`**=0.05.**

q	plars	RF	XGB	KNN	DNN
50%	0.05	14.0%	14.0%	20.0%	1988.0%
80%	0.1	16.0%	14.0%	21.0%	1155.0%
95%	0.16	25.0%	20.0%	22.0%	738.0%
99%	0.27	76.0%	64.0%	24.0%	975.0%

Results for `lorentz` **with $\mathbf{z} = \mathbf{x_2}$, $\sigma_\mathbf{p} = \mathbf{0.1}$ and** `noise`**=0.05.**

q	plars	RF	XGB	KNN	DNN
50%	0.05	13.0%	13.0%	20.0%	2017.0%
80%	0.09	11.0%	13.0%	19.0%	1300.0%
95%	0.16	13.0%	20.0%	18.0%	836.0%
99%	0.28	33.0%	62.0%	41.0%	793.0%

Fig. 4. Partial estimation results (on unseen test data) for the observation target $z = x_2$ of `Lorentz` system with different levels of parametric dispersion and measurement noise.

Results

Results for `ETC` **with** $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = \mathbf{0.}$ **and** `noise`**=0.0.**

q	plars	RF	XGB	KNN	DNN
50%					
80%					
95%					
99%					

$$100 \times \left[\frac{p_q^{\texttt{algo}} - p_q^{\texttt{plars}}}{p^{\texttt{plars}} + \epsilon}\right] \quad ; \quad \epsilon = 0.001$$

$$p_q^{\texttt{plars}} := \frac{\texttt{percentile}(|z - \hat{z}|, q)}{\texttt{median}(|z|)}$$

Results for `lorentz` **with** $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = \mathbf{0.05}$ **and** `noise`**=0.025.**

q	plars	RF	XGB	KNN	DNN
50%	0.03	59.0%	28.0%	52.0%	3383.0%
80%	0.06	60.0%	34.0%	53.0%	1950.0%
95%	0.1	63.0%	71.0%	63.0%	1190.0%
99%	0.18	121.0%	145.0%	75.0%	1390.0%

Results for `lorentz` **with** $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = \mathbf{0.1}$ **and** `noise`**=0.025.**

q	plars	RF	XGB	KNN	DNN
50%	0.03	46.0%	31.0%	62.0%	3581.0%
80%	0.05	49.0%	38.0%	64.0%	2340.0%
95%	0.1	59.0%	61.0%	73.0%	1458.0%
99%	0.18	120.0%	130.0%	104.0%	1370.0%

Results for `lorentz` **with** $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = \mathbf{0.05}$ **and** `noise`**=0.05.**

q	plars	RF	XGB	KNN	DNN
50%	0.05	14.0%	14.0%	20.0%	1988.0%
80%	0.1	16.0%	14.0%	21.0%	1155.0%
95%	0.16	25.0%	20.0%	22.0%	738.0%
99%	0.27	76.0%	64.0%	24.0%	975.0%

Results for `lorentz` **with** $\mathbf{z = x_2}$, $\sigma_\mathbf{p} = \mathbf{0.1}$ **and** `noise`**=0.05.**

q	plars	RF	XGB	KNN	DNN
50%	0.05	13.0%	13.0%	20.0%	2017.0%
80%	0.09	11.0%	13.0%	19.0%	1300.0%
95%	0.16	13.0%	20.0%	18.0%	836.0%
99%	0.28	33.0%	62.0%	41.0%	793.0%

Fig. 4. Partial estimation results (on unseen test data) for the observation target $z = x_2$ of `Lorentz` system with different levels of parametric dispersion and measurement noise.

Dataset available @:

MA. Dataset for nonlinear state estimator design. 10.34740/KAGGLE/DSV/10016280

Full details available @:

MA. Partial state estimator design for Nonlinear Uncertain System. In Nonlinear control of Uncertain Systems. **Springer Nature** 2026.

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Sparse models derivation is crucial for learning with limited data.

Since, successful extension of the the current investigation to piece-wise polynomial relationships.

Take-Home Messages

Learning might be a way to explicitly build **partial stochastic estimator** in the presence of uncertainties with known dispersion.

Sparse models derivation is crucial for learning with **limited data**.

Since, successful extension of the the current investigation to **piece-wise polynomial** relationships.

Questions?