Comparison of primal and a dual decomposition for distributed MPC in smart districts

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SmartGridComm 2014, Venice - Italy, 4th November 2014

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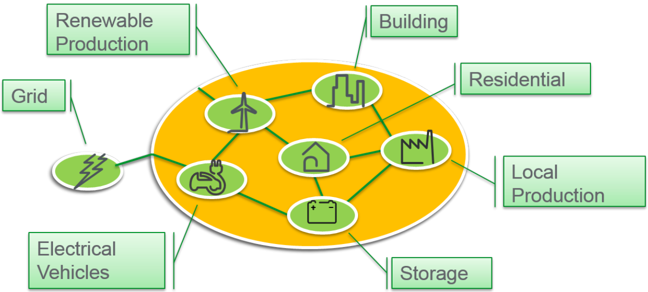
Smart District system



- District containing local production, storage and consumers
- Common grid connection with possible constraints (power limitation,...)
- Modern communication infrastructure allowing the implementation of advanced control strategies

Objective

Use predictions & the flexibilities of the district actors in order to...

- reduce energy consumption from grid
- reduce peak consumption
- reduce CO_2 emissions
- reduce the energy bill

Available flexibilities:

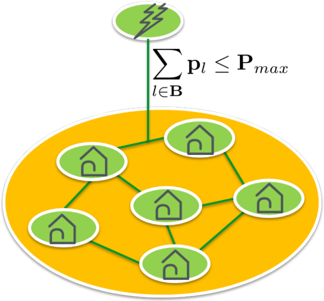
- Storage capacities (batteries, hot water tanks, EV,...)
- Inertias (building temperature,...)
- Controlled local production (CHP, gaz turbine,...)

Resource sharing problem

- Optimize the service of dynamically uncoupled sub systems which share a globally limited resource

In the smart district context:

- several buildings with no coupling between each other
- shared connection to grid with a global power limit



Local building MPC:

- bilinear single zone building models

$$x^+ = A \cdot x + [B(y,w)] \cdot u + F \cdot w$$
$$y = C \cdot x + [D(w)] \cdot u \qquad (1)$$

  Note that $B(y,w)$ is affine in y and w.

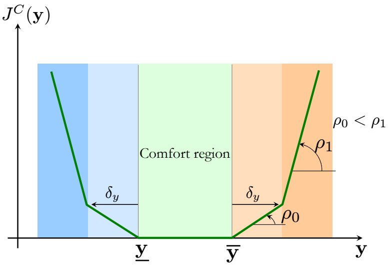
- results in a nonlinear MPC problem which is solved using a fixed-point algorithm (see [?] for a detailed convergence discussion of the approach)

$$\underset{\mathbf{u}\in\mathbf{U}}{\textbf{Minimize}} \ J := J^E(\mathbf{p}) + J^C(\mathbf{y})$$

where:

- $\mathbf{p}$ is the power consumption
- J^C is the discomfort criterion
- J^E is the energy criterion

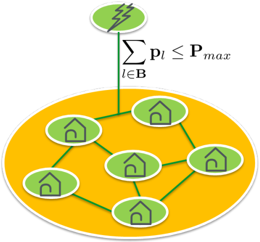
Discomfort criterion J^C:



District MPC problem

$$\underset{\mathbf{u}_1,\ldots,\mathbf{u}_{n_B}}{\text{Minimize}} \quad \sum_{l\in\mathbf{B}} \mathbf{J}_l(\mathbf{p}_l, \mathbf{y}_l) \qquad (2a)$$

$$\sum_{l\in\mathbf{B}} \mathbf{p}_l \leq \mathbf{P}_{max} \qquad (2b)$$



- The local MPC problems are coupled through the limited shared resource (power)
- Distributing the centralized MPC problem is desirable due to its large-scale character
- No easy way to decouple the system because of coupling constraint (2b)

Decomposition methods

Two hierarchical DMPC methods are compared:

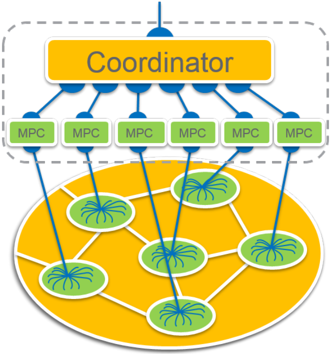
- Method 1 which is based on a primal decomposition of the centralized problem
- Method 2 which is based on a dual decomposition of the centralized problem

Primal decomposition:

- The coordinator directly determines the optimal power allocation for each sub system while respecting the global power limit

Dual decomposition:

- In order to respect the global power limit the coordinator imposes a "virtual tariff" on the limited resources such that the local MPC controllers adapat their consumption behavior in the desired way.



Principle:

- Iterative process between coordinator and local MPC controllers in both cases
- Convergence to initial centralized solution

Comparison of the two decomposition methods

Primal decomposition:

- Feasible solution at each iteration
- Number of decision variables at coordinator level increases importantly with the number of subsystems
- The sub systems send their gradient w.r.t. their local resource limit to the coordinator

Dual decomposition:

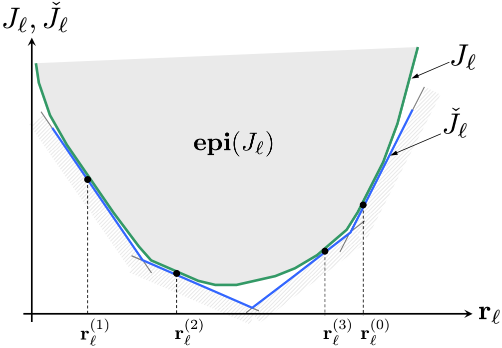
- Feasibility only after convergence
- Number of decision variables at coordinator level does NOT increase with the number of subsystems
- The sub systems send their resource consumption to the coordinator

Equivalence of the solutions:

- If strong duality holds both approaches result in an equally good solution
- In fact, the dual solution provides a lower bound on the primal solution
- The duality gap serves as an indicator of the equivalence: $f^* - g^* \geq 0$ where f^*, g^* are the optimal primal and dual objective values

The bundle method

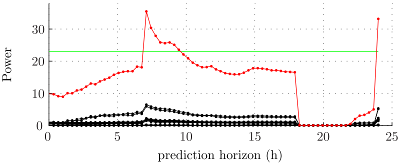
- Iteratively build a cutting plane approximation of the sub systems' objective functions
- Small number of iterations due to memory of the bundles after shifting the horizon (5-10 iterations)



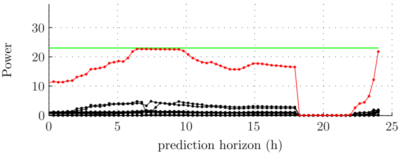
District power consumption after convergence

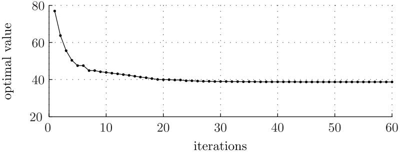
- 20 buildings, severe power limit (23kW), initial point far from optimum

uncoordinated:

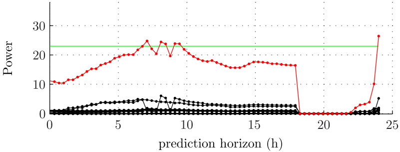


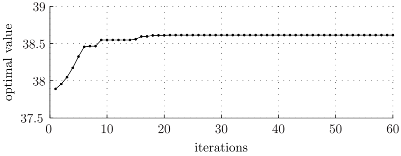
primal decomposition:





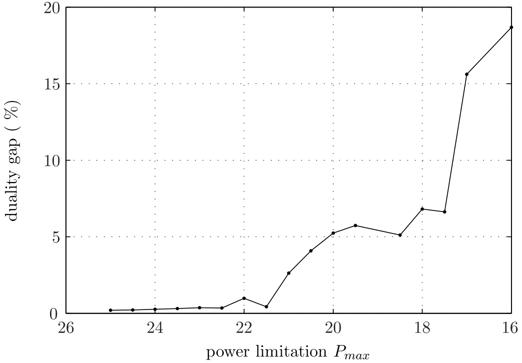
dual decomposition:





Duality gap

- District power limit becoming more and more severe
- Duality gap plotted as a function of the power limit



Note: Additionally to the increasing duality gap, also the global power limit is not respected anymore with the dual decomposition approach for very severe power limits.

Computation times

- Number of decision variables of the centralized problem: ≈ 20000
- Building MPC problems can be solved in parallel
- Time to solve one building MPC problem: $t_{local} = 75ms$
- Time to solve the coordinator problem: $t_{Coordinator} = 79ms$
- Around 10 iterations required in closed loop to achieve optimality for both approaches

Resulting total time to solve the district problem

$$t_{total} = n_{iter} \cdot (t_{local} + t_{Coordinator}) = 10 \cdot (75[ms] + 79[ms]) = 1.54[s]$$