# On Adaptative Measurement Inclusion Rate In Real-Time Moving-Horizon Observers

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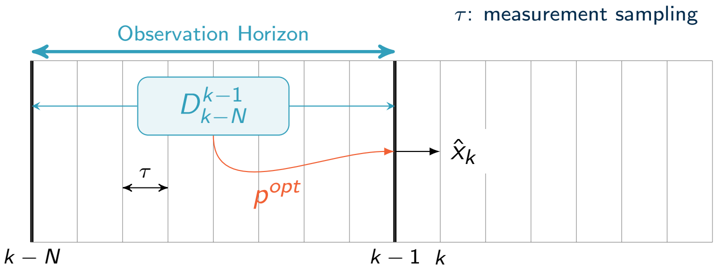
# Outline

Problem Statement

Sketch of the solution

Illustrtative example

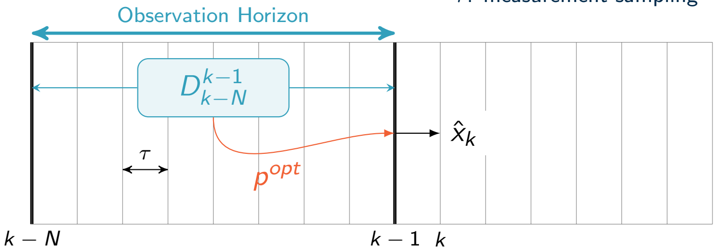
# The measurement inclusion rate




$$
\begin{aligned}
p^{opt} &= \underset{p\in\mathbb{P}}{\text{argmin}}\, J(p, D_{k-N}^{k-1}) \\
\hat{x}_k &= F(p^{opt}, \mathbf{u}_{k-1}^{k})
\end{aligned}
$$

# The measurement inclusion rate

$\tau$: measurement sampling

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- typical choices for $p$:
  - state value at $k-N$
  - whole state trajectory

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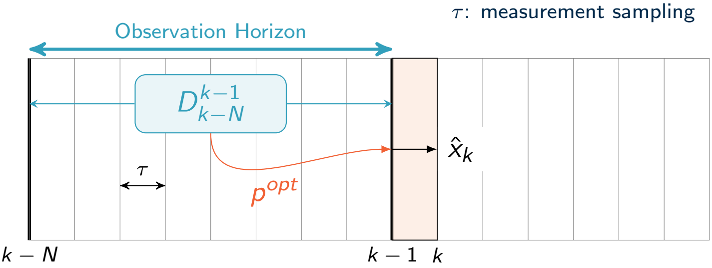


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- solution need less than $\tau$
- Measurement are included once available
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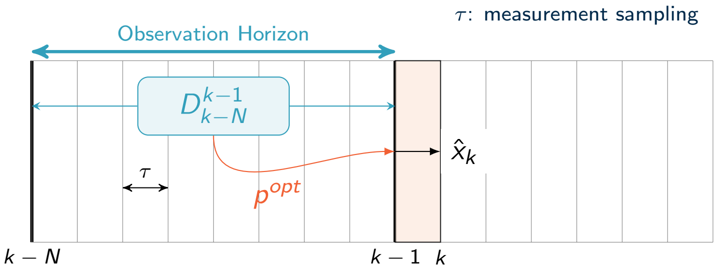


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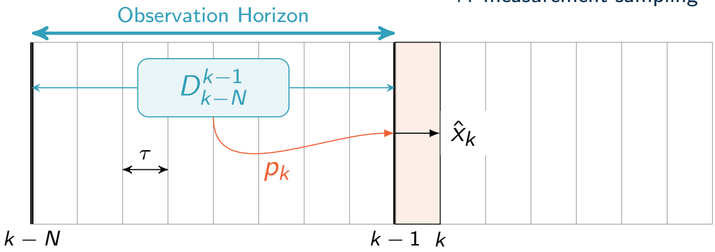


$$p_k = \mathcal{S}^{(q)}(p^*_{k-1}, D_{k-N}^{k-1})$$
$$\hat{x}_k = F(p_k, \mathbf{u}_{k-1}^k)$$

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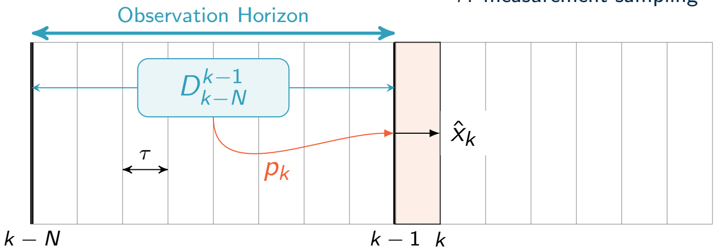


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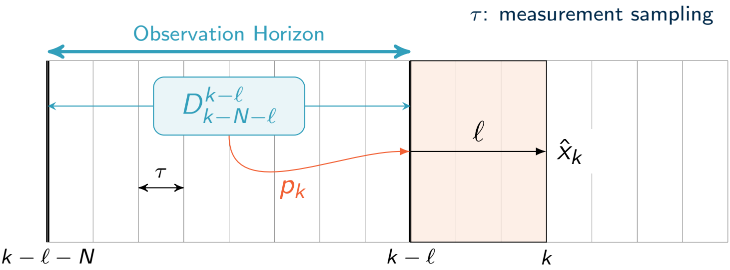
$q$: nb of iterations during $\tau$:

$$q = \lfloor \frac{\tau}{\tau_c} \rfloor$$

$\tau_c$ time for a single iteration.

# The measurement inclusion rate




$$p_k = \mathcal{S}^{(q(\ell))}(p^*(k-\ell), D_{k-\ell-N}^{k-\ell})$$

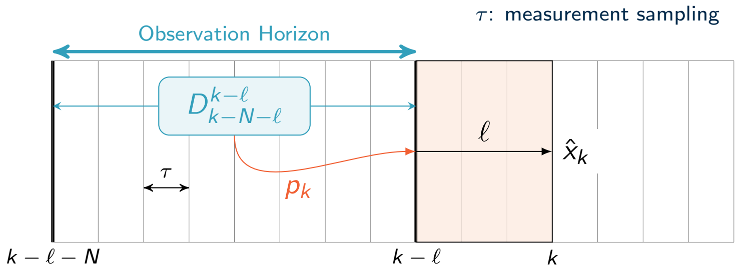
$$\hat{x}_k = F(p_k, \mathbf{u}_{k-\ell}^{k})$$

$q(\ell)$: nb of iterations during $\ell\tau$:

$$q(\ell) = \lfloor \frac{\ell \times \tau}{\tau_c} \rfloor$$

$\tau_c$ time for a single iteration.

# The measurement inclusion rate




**NOTA**

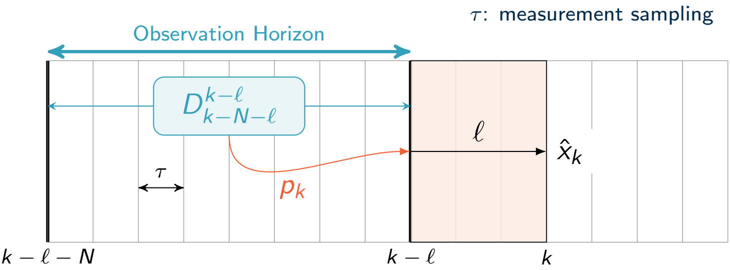
$\ell$ (or $q$) determines the rate with which the new measurement are included.

$q(\ell)$: nb of iterations during $\ell\tau$:

$$q(\ell) = \lfloor \frac{\ell \times \tau}{\tau_c} \rfloor$$

$\tau_c$ time for a single iteration.

# The measurement inclusion rate




**NOTA**

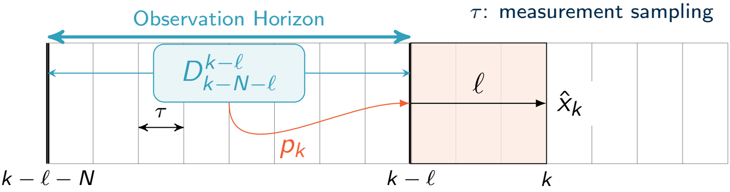
$\ell$ (or $q$) determines how long the solver works on an unchanged cost function.

$q(\ell)$: nb of iterations during $\ell\tau$:

$$q(\ell) = \lfloor \frac{\ell \times \tau}{\tau_c} \rfloor$$

$\tau_c$ time for a single iteration.

# The trade-off



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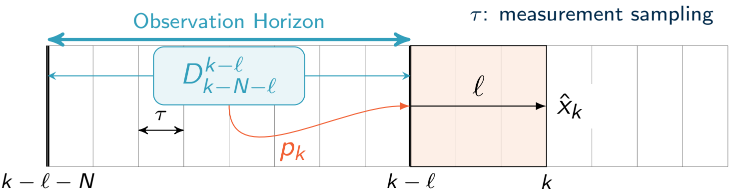


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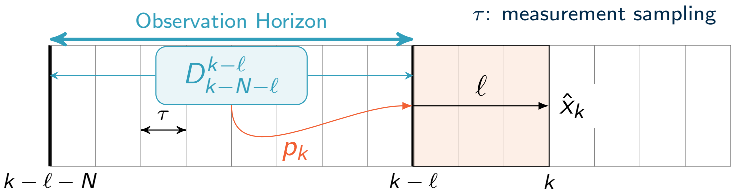


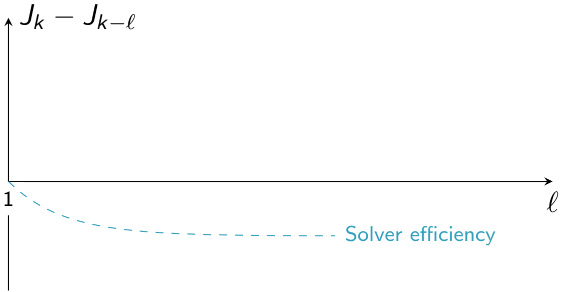

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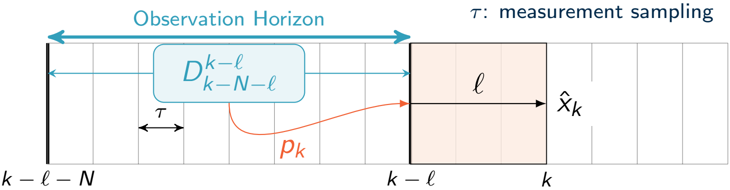


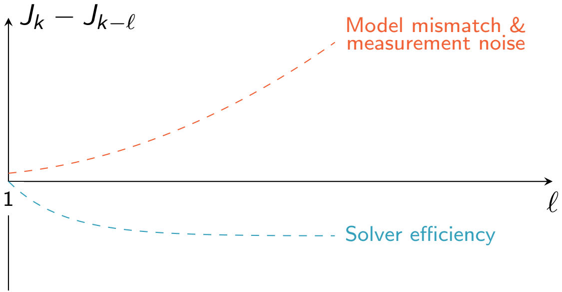

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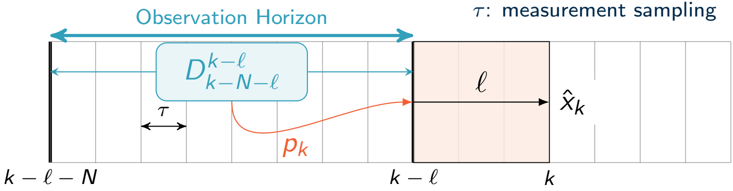


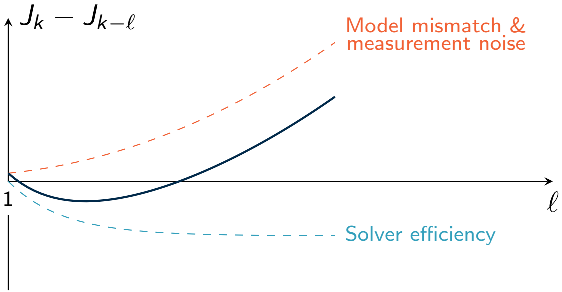

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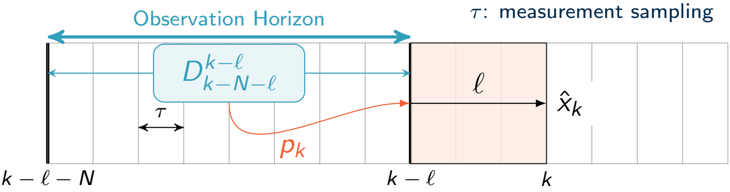


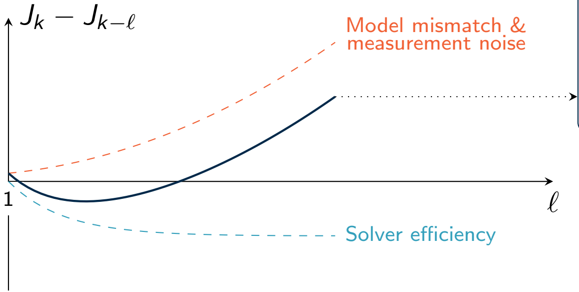


$\ell = 1$ is not necessarily the optimal choice.

The optimal choice is context dependent.

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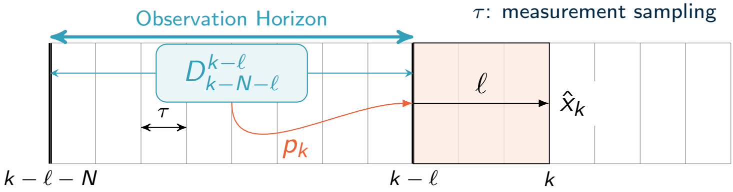


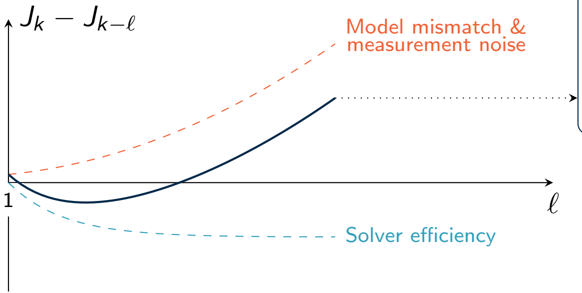


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The optimal choice is context dependent.

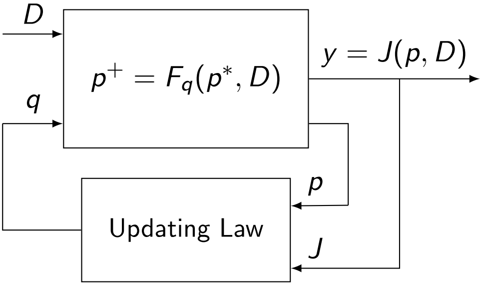
on-line adaptation of $\ell$ (or $q$)

## Sketch of the solution

- Dual of NMPC version

  [MA, Monitoring Control Updating Period In Fast Gradient-Based NMPC. Proceedings of the European Control Conference, July 17-19, Zurich, 2013].

- (control updating period) $\leftrightarrow$ (measurement inclusion rate)




- Minimize the response time of $y$
- Gradient descent on $q \in [q_{min}, q_{max}]$
- 5($\pm$), 5($\times$), 6($\div$), 1 log
- See paper for details

## Illustrative example: problem data

**System & measurement**

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -ax_1 + (1 - ux_3x_1^2)x_2\\ \dot{x}_3 &= 0\\ y &= x_1 + \nu\end{aligned}$$

- $\tau = 2\ ms$
- $\mathcal{S}$: Fast Gradient ($\tau_c = 100\ \mu sec.$) [Nesterov 1983, 2004]
- $J := \sum_{i=0}^{N} |\hat{y}_{k+i}(p) - y_{k+i}|^2 + \rho\|p - p^*_{k-1}\|^2$, ($\rho = 0.01$)
- $u(t) = 1 - 0.5\cos(2t)$
- $\sigma_\nu = 0.03$, $q \in [20, 1000]$ ($\ell \in [1, 50]$)
- Increment size for gradient method $\delta = 10$

# Illustrative example: problem data

**System & measurement**

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a x_1 + (1 - u x_3 x_1^2) x_2$$
$$\dot{x}_3 = 0$$
$$y = x_1 + \nu$$

$$q = \frac{\ell \tau}{\tau_c} = 20\ell$$

- $\tau = 2\ ms$
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- $J := \sum_{i=0}^{N} |\hat{y}_{k+i}(p) - y_{k+i}|^2 + \rho \|p - p^*_{k-1}\|^2$, ($\rho = 0.01$)
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# Illustrative example: validation scenarios

**two groups of simulations**

- $\hat{a} = a = 10$ (no uncertainty)
- $\hat{a} = 7,\ a = 10$ (model mismatch)

**For each case**, 5 settings:

| | | |
|---|---|---|
| (s=1) | $q = 20$ | No updating. |
| (s=2) | $q = 50$ | No updating. |
| (s=3) | $q = 100$ | No updating. |
| (s=4) | $q = 300$ | No updating. |
| (s=5) | $q(0) = 20$ | with updating. |

50 different $\hat{x}^0$ are randomly generated

$$\hat{x}_i^0 \in [0.2x_i^0, 2x_i^0]$$

For each triplet $(\hat{a}, s, \hat{x}^0)$, a simulation is performed over $N_{sim} = 2000$ periods to obtain:

$$\left\{J^{(s)}(k, \hat{x}^0)\right\}_{k=1}^{N_{sim}-N}$$

**Normalized indicator**:

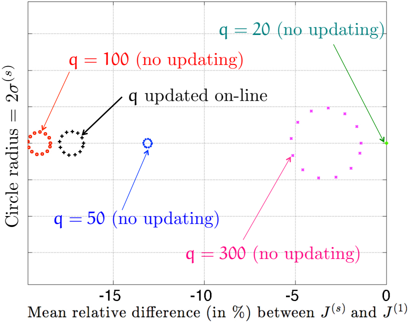
$$m^{(s)} \quad := \quad mean_{k,\hat{x}^0}\left[\frac{J^{(s)}(k,\hat{x}^0) - J^{(1)}(k,\hat{x}^0)}{J^{(1)}(k,\hat{x}^0)}\right]$$

$$\sigma^{(s)} \quad := \quad var_{k,\hat{x}^0}\left[\frac{J^{(s)}(k,\hat{x}^0) - J^{(1)}(k,\hat{x}^0)}{J^{(1)}(k,\hat{x}^0)}\right]$$

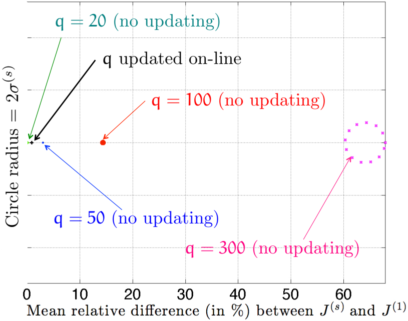
card$= 50 \times (2000 - 200) = 90.000$

# Illustrative example: results

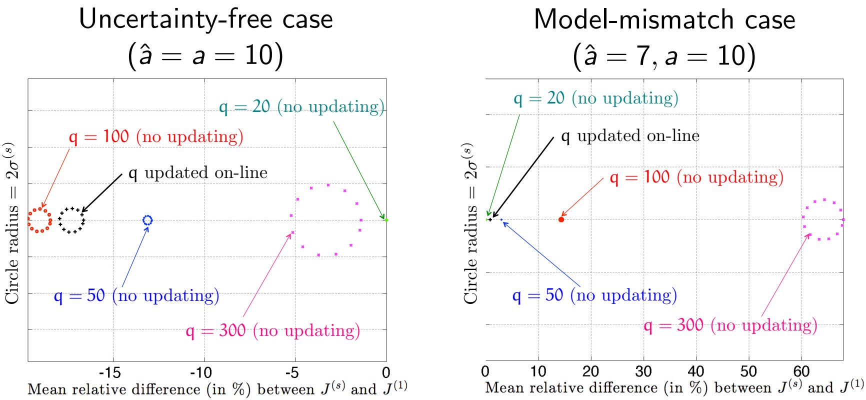
## Uncertainty-free case ($\hat{a} = a = 10$)




## Model-mismatch case ($\hat{a} = 7, a = 10$)



## Illustrative example: results




- The optimal choice is context dependent
- Updating leads to *near optimal* values
- Particularly suitable for time varying context

## Conclusion

- Extension of successful ideas from RT-NMPC to RT-MHO
- Cheap updating algorithm
- Generic layer that can be used with any favorite $\mathcal{S}$
- Needs further validation for different kinds of observer cost