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Technical communique

Stability proof for nonlinear MPC design using monotonically increasing weighting profiles without terminal constraints[☆]

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ABSTRACT

In this note, a new formulation of Model Predictive Control (MPC) framework with no stability-related terminal constraint is proposed and its stability is proved under mild standard assumptions. The novelty in the formulation lies in the use of time-varying monotonically increasing stage cost penalty. The main result is that the 0-reachability prediction horizon can always be made stabilizing without any terminal constraints provided that the increasing rate of the penalty is made sufficiently high. Moreover, it is shown through an illustrative example that the time varying penalty may improve the resulting closed-loop performance computed with the original stage-cost when compared to the traditional MPC formulation with final constraint on the state.

1. Introduction

In the majority of NMPC formulations, the stability of the closed-loop implies the use of terminal constraints on the state. In the early formulations (Alamir & Bornard, 1994; Keerthi & Gilbert, 1988; Mayne & Michalska, 1990), a strong point-wise equality constraint on the state was introduced. This constraint is imposed at the end of the prediction horizon, namely N-steps ahead where N is such that the targeted state is N-step reachable. Since, relaxations were introduced through the combined use of terminal set constraint and appropriate terminal penalty. The many different ways to choose these two items were unified in Mayne, Rawlings, Rao, and Scokaert (2000) where it has been shown that the terminal set should be controlled-invariant under some *local* feedback control that makes the terminal penalty a control-Lyapunov function inside the terminal set. The difficulty in computing the terminal set and the associated Lyapunov function for general nonlinear systems remains quite dissuasive in real-life applications. On the other hand, it has been shown quite early (Alamir & Bornard, 1995) that provable stability can be obtained without terminal stability-related constraint by using *sufficiently long* prediction horizon (Grimm, Messina, Tuna, & Teel, 2005; Jadbabaie & Hauser, 2005). More recent results followed, [see Boccia, Grüne, & Worthmann, 2014; Grüne & Pannek, 2011; Grüne, Pannek, Seehafer, & Worthmann, 2010 and the references therein] where deeper analysis is obtained regarding this fact. However, the underlying argument remained that with sufficiently long prediction horizon, the optimal decisions necessarily lead to open-loop trajectories with terminal appropriate properties.

Another family of stabilizing formulations without terminal constraint are those based on the contraction property (Alamir, 2007, 2017; Kothare, de Oliveira, & Morari, 2000). These formulations are quite attractive in terms of the minimal necessary prediction horizon length although they sometimes involve rather non conventional implementation forms and/or Lyapunov functions.

This note proposes a flash-back towards the early N-reachability related formulations with the exception that no stability-related terminal constraint is used. Instead, a non uniform (in time) penalty is used in the definition of the stage cost. This obviously strengthens the weight on the tail of the prediction horizon leading to similar effects as those induced by the infinite (or sufficiently long) horizon costs. The idea might seem straightforward and it is probably so. Nevertheless, this technical note gives the formal proof of the stability result. To say it shortly, the proposed formulation inherits the combined advantages of early formulations (finite, short and stabilizing horizon) and the infinite horizon formulations (absence of final stability-related constraint).

To summarize, the novelty and the relevance of the proposed formulation lie in the following facts:
(1) The proposed formulation uses the same standard assumptions (N-steps reachability and existence of local stability components) while avoiding the explicit use of the local components or any stability related terminal constraint.
(2) By achieving the above, the computational complexity for on-line nonlinear MPC is drastically reduced thanks to the removal

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of the terminal stability constraints while keeping the original complexity in the computation of the stage cost.
(3) The two advantages mentioned above are not paid through a drop in the original performance index, or at least no evidence exist for such a systematic effect. The illustrative example proposed in Section 5 even suggests that the proposed formulation might induce an improved closed-loop performance expressed in terms of the original cost, especially for short horizons which are the most interesting in the case of on-line real-time MPC implementation. This may be understood by the fact that the terminal stability-related constraints induce higher distance to the optimal solution than the change in the stage cost that is induced by the non-uniform penalty profile.
(4) On the top of the previous advantages, the absence of the need for the terminal constraints corresponds to an easier MPC design as the computation of the terminal set and the associated terminal weight can be cumbersome for general nonlinear systems.

The paper is organized as follows: The problem is stated in Section 2. The necessary assumptions are given in Section 3 before the main result is given in Section 4. Section 5 gives an example that illustrates the items discussed above.

It is worth underlying that while only control-related constraints are included in the formulation of the present note, the result of this note can easily be extended to the case where state constraints are present. This would, however, add useless (and quite standard) technicalities that may hide the main underlying arguments. Remark 3 gives however the main changes that need to be incorporated to handle state constraints.

2. Problem statement

Let us consider general nonlinear systems of the form:

$$x_{k+1} = f(x_k, u_k) \tag{1}$$

where $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^{n_u}$ represent the state and the control vectors respectively at instant k. It is assumed that $f(0, 0) = 0$ and that the control objective is to stabilize the steady state $x = 0$.

Consider control profiles $\boldsymbol{u} := (u_0, u_1, \dots, u_{N-1})$ defined over a prediction horizon of length N. Denote the corresponding state trajectory starting from x_k by $\boldsymbol{x}_i^{\boldsymbol{u}}(x_k)$ for $i = 0, \dots, N$, namely:

$$\boldsymbol{x}_0^{\boldsymbol{u}}(x_k) = x_k \tag{2}$$

$$\boldsymbol{x}_i^{\boldsymbol{u}}(x_k) = f(\boldsymbol{x}_{i-1}^{\boldsymbol{u}}(x_k), u_{i-1}) \quad \text{for } i \in \{1, \dots, N\}. \tag{3}$$

Based on the trajectories $\boldsymbol{u}$ and $\boldsymbol{x}^{\boldsymbol{u}}(x_k)$, let us consider a cost function of the form:

$$J_m(\boldsymbol{u}|x_k) := \sum_{i=1}^{N} (i/N)^m \ell(\boldsymbol{x}_i^{\boldsymbol{u}}(x_k)) \tag{4}$$

for some integer $m \in \mathbb{N}$. This cost enables the following optimization problem to be defined for any compact set $\mathbb{U} \subset \mathbb{R}^{Nn_u}$ of admissible control profiles:

$$\mathcal{P}_m(x_k): \quad \min_{\boldsymbol{u}\in\mathbb{U}} J_m(\boldsymbol{u}|x_k). \tag{5}$$

Let us denote a solution to $\mathcal{P}(x_k)$ (if any) by $\boldsymbol{u}^*(x_k, m)$ and the corresponding optimal cost $J_m^*(x_k)$.

The aim of this note is to investigate the conditions under which $x = 0$ is an asymptotically stable equilibrium for the closed-loop system given by:

$$x_{k+1} = f(x_k, \boldsymbol{u}^*(x_k, m)). \tag{6}$$

2.1. Notation

In what follows, the subset $B_\ell(\rho) \subset \mathbb{R}^n$ denotes the ρ-level set of ℓ, namely $B_\ell(\rho) := \{x \in \mathbb{R}^n \mid \ell(x) \le \rho\}$.

3. Statement of the required assumptions

The first assumption is a standard N-step reachability condition of the targeted state $x = 0$. This Assumption is commonly used in the first provably-stable formulations (Alamir & Bornard, 1994; Keerthi & Gilbert, 1988; Mayne & Michalska, 1990):

Assumption 1. The maps f and ℓ are continuous and ℓ is positive definite. Moreover, there is a compact set $\mathbb{X}_N$ such that for all $x \in \mathbb{X}_N$, the set defined by:

$$\mathbb{U}_{x\to 0} := \left\{\boldsymbol{u} \in \mathbb{U} \quad \text{s.t} \quad \boldsymbol{x}_N^{\boldsymbol{u}}(x) = 0\right\} \tag{7}$$

is not empty.

The second assumption is a local control-invariance property that is assumed in the neighborhood of the origin.

Assumption 2. There exists $\bar{\rho} > 0$ such that
$\forall x \in B_\ell(\bar{\rho}), \exists u^+$ s.t,

$$\ell(f(x, u^+)) - \ell(x) \le -q(x) \tag{8}$$

for some positive definite function q satisfying:

$$q(x) \ge \gamma \ell(x) \tag{9}$$

for some $\gamma > 0$ and for all $x \in \mathbb{X}_N$.

This is again a rather standard assumption since (8) simply means that ℓ is a **local** control-Lyapunov function with decrease rate described by q. Moreover, the inequality (9) generically holds for a large choice of ℓ and q including positive definite quadratic forms. More precisely, if $\ell(x) = x^T Q_\ell x$ and $q = x^T Q_q x$ then $\gamma := \frac{\lambda_{min}(Q_q)}{\lambda_{max}(Q_\ell)}$ satisfies the condition (9).

4. Main results

Lemma 1. *Under Assumption 1, for all $x \in \mathbb{X}_N$, one has:*

$$\ell(\boldsymbol{x}_N^{\boldsymbol{u}^*(x,m)}(x)) \le \eta \cdot c^m \quad \text{where } c := \frac{N-1}{N} < 1 \tag{10}$$

for some bounded $\eta > 0$.

Proof. Take $\boldsymbol{u}^0 \in \mathbb{U}_{x\to 0}$ which is possible (since $x \in \mathbb{X}_N$) by virtue of Assumption 1. By the definition of optimality one has:

$$J_m^*(x) \le J_m(\boldsymbol{u}^0|x) \le \sum_{i=1}^{N} (i/N)^m \ell(\boldsymbol{x}_i^{\boldsymbol{u}^0}(x))$$

and since $\ell(\boldsymbol{x}_N^{\boldsymbol{u}^0}(x)) = 0$ by assumption, the last term can be removed to get:

$$J_m^*(x) \le \sum_{i=1}^{N-1} (i/N)^m \ell(\boldsymbol{x}_i^{\boldsymbol{u}^0}(x))$$

$$\le \Big(\frac{N-1}{N}\Big)^m \sum_{i=1}^{N-1} \ell(\boldsymbol{x}_i^{\boldsymbol{u}^0}(x)).$$

This obviously gives the result if η is defined s.t:

$$\eta \ge \sup_{(x,\boldsymbol{u})\in\mathbb{X}_N\times\mathbb{U}^N} \sum_{i=1}^{N-1} \ell(\boldsymbol{x}_i^{\boldsymbol{u}}(x)) \tag{11}$$

which is obviously well defined and bounded by continuity of f, ℓ and the fact that $\mathbb{X}_N \times \mathbb{U}^N$ is a compact set. □

Proposition 2. *Under Assumptions 1* and *2, the targeted state $x = 0$ is asymptotically stable for the closed-loop dynamics (6) for all initial state $x \in \mathbb{X}_N$.*

Proof. Let us shortly denote the optimal profile at instant k by $\boldsymbol{u}^* := \boldsymbol{u}^*(x_k, m)$. At instant $k+1$, consider the candidate control profile $\tilde{\boldsymbol{u}}_k$ defined by:

$$\tilde{\boldsymbol{u}} := \left(\boldsymbol{u}_1^*, \dots, \boldsymbol{u}_{N-1}^*, \bar{u}\right) \tag{12}$$

where $\bar{u} \in \mathbb{U}$ is defined by:

$$\bar{u} := \arg\min_{u\in\mathbb{U}} \quad \ell\Big(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), u)\Big). \tag{13}$$

As a candidate solution to $\mathcal{P}_m(x_{k+1})$, $\tilde{\boldsymbol{u}}$ corresponds to a cost function satisfying:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) + \sum_{i=1}^{N-1}(i/N)^m \ell(\boldsymbol{x}_{i+1}^{\boldsymbol{u}^*}(x_k))$$

which can be rewritten using the change of indices $j = i+1$ as follows:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) + \sum_{j=2}^{N}(\frac{j-1}{N})^m \ell(\boldsymbol{x}_j^{\boldsymbol{u}^*}(x_k)). \tag{14}$$

Now for the sake of readability, let us use the following compact notation $\ell_j^*(x_k) := \ell(\boldsymbol{x}_j^{\boldsymbol{u}^*}(x_k))$. With this notation, Eq. (14) becomes:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) + \sum_{j=2}^{N}(\frac{j-1}{N})^m \ell_j^*(x_k)$$

using straightforward neutral operations, it comes that:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) + \sum_{j=2}^{N}(\frac{j-1}{j})^m(\frac{j}{N})^m \ell_j^*(x_k)$$

and by adding and removing the same terms:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) + \sum_{j=2}^{N}\Big[(\frac{j-1}{j})^m - 1\Big](\frac{j}{N})^m \ell_j^*(x_k) + \sum_{j=2}^{N}(\frac{j}{N})^m \ell_j^*(x_k). \tag{15}$$

But note that the last term in (15) satisfies:

$$\sum_{j=2}^{N}(\frac{j}{N})^m \ell_j^*(x_k) = J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k). \tag{16}$$

Using this last equation in (15) gives:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) = J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) + \tag{17}$$
$$-\sum_{j=2}^{N}\Big[1-(\frac{j-1}{j})^m\Big](\frac{j}{N})^m \ell_j^*(x_k) + \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right)$$

and since for all $j \in \{2, \dots, N\}$, one has:

$$\Big[1-(\frac{j-1}{j})^m\Big] \geq \Big[1-(\frac{N-1}{N})^m\Big] =: \psi(m) \tag{18}$$

Eq. (17) implies:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) \leq J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) - \psi(m)\sum_{j=2}^{N}(\frac{j}{N})^m \ell_j^*(x_k) + \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right) \tag{19}$$

and keeping only the last term of the sum in the first term of the second line, one obtains:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) \leq J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) - \psi(m)\ell_N^*(x_k) + \ell\left(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\right). \tag{20}$$

Now according to Lemma 1, $\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k) \in B_\ell(\eta c^m)$ which, together with Assumption 2 implies that for sufficiently high m, one has:

$$\eta c^m \leq \bar{\rho} \tag{21}$$

where $\bar{\rho}$ is the positive real invoked in Assumption 2. This means that (8) holds for $\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k)$, namely:

$$\ell\Big(f(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k), \bar{u})\Big) \leq \ell\Big(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k)\Big) - q(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k)) \leq \ell_N^*(x_k) - q(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k)) \tag{22}$$

using this last inequality in (20) leads to:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) \leq J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) + (1-\psi(m))\ell_N^*(x_k) - q(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k)) \tag{23}$$

which by the definition of $\ell_N^*(x_k) := \ell(\boldsymbol{x}_N^{\boldsymbol{u}^*}(x_k))$ and by (9) of Assumption 2 implies:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) \leq J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) - (\psi(m) - 1 + \gamma)\ell_N^*(x_k) \tag{24}$$

and therefore, as $\psi(m) \to 1$ when $m \to \infty$, there is a finite m beyond which one has:

$$J_m(\tilde{\boldsymbol{u}}|x_{k+1}) \leq J_m^*(x_k) - \frac{1}{N^m}\ell_1^*(x_k) - \frac{\gamma}{2}\ell_N^*(x_k). \tag{25}$$

Finally, recalling that $\ell_1^*(x_k)$ is nothing but $\ell(x_{k+1})$ obviously ends the proof. □

Remark 3 (*Incorporating State Constraints*)**.** The main changes leading to the incorporation of state constraints are:

(1) The definition of the subset $\mathbb{X}_N$ should incorporate the constraints satisfaction.
(2) The definition of the local set $B_{\bar{\rho}}$ should incorporate constraint satisfaction. This guarantee recursive feasibility.

Apart from these changes, the proof of the main result is rigorously the same.

Remark 4 (*Effect on Closed-loop Performance*)**.** One might rightly ask whether the use of the non uniform penalty profile affects the physical relevance of the stage cost when engineering problems are considered. Although the paper does not give a definitive and rigorous answer to this question, it can be argued that provably stable traditional formulations introduce also a kind of sub-optimality by restricting the admissible domain through the terminal constraints and by adding a terminal penalty weighting. Whether these changes impact the closed-loop performance (expressed in terms of the original stage cost) more or less than the non-uniform penalty proposed here is likely to be problem and parameters dependent. The simulations of Section 5 give at least an example where the proposed formulation improves the closed-loop performance, especially for short prediction horizons.

5. Illustrative example

Although the proposed formulation's target is the design of computationally efficient MPC for nonlinear systems, we consider here a simple linear system in order to indisputably quantify the behavior of the stability/performance indicators when using the proposed formulation.[1] The qualitative conclusions that one draws from this example obviously hold for nonlinear systems. Let us then consider the following triple integrator system given by:

$$\dot{x}_1 = x_2 \ , \ \dot{x}_2 = x_3 \ , \ \dot{x}_3 = v \tag{26}$$

that we want to control using MPC with a stage cost involving the penalty matrices $Q \in \mathbb{R}^{3\times 3}$ and $R \in \mathbb{R}$. Note that since the formulation of the present paper allows only a stage cost that depends on the system's state (and not the control), we extend the state vector by considering the control $v = x_4$ as the fourth state while the new control becomes the derivative of the original control $\dot{x}_4 = u$. By doing so, the same stage cost can be put in the standard form $\ell(x) = x^T\bar{Q}x$ where $\bar{Q} := diag(Q, R)$. This can be now used in the cost function (4) to give the optimal feedback of the proposed formulation in the form

$$\boldsymbol{u}^*(x, m) := -K^{(N,m)}(x - x_d) \tag{27}$$

where $K^{(N,m)} \in \mathbb{R}^{1\times 4}$ can be computed by expressing the fact that (4) is a quadratic function in the decision variable $\tilde{\boldsymbol{u}} \in \mathbb{R}^N$.

Fig. 1 shows the spectrum of the closed-loop matrix for different values of the pair (N, m). Obviously, if no increasing penalty is used ($m = 0$), the minimal prediction horizon length to achieve stability (without terminal constraint) would be $N = 20$. When the proposed formulation is used, stability can be achieved with a prediction horizon as short as $N = 3$ provided that appropriate corresponding power m is used ($m = 10$ for instance).

On the other hand the traditional MPC with final equality constraints can be obtained by minimizing the original cost function [the same as (4) in which $m = 0$ is used] under the terminal constraint $\boldsymbol{x}_N^{\tilde{\boldsymbol{u}}}(x_k) = x_d(k)$. The optimal feedback is denoted by $\boldsymbol{u}^{cstr}(x)$.

The closed-loop is simulated under the two controllers with the desired output profile y_d depicted in Fig. 2. This results in the desired state profile $x_d(k) = (y_d(k), 0, 0, 0)^T$. The two corresponding closed-loop costs (starting from $x_0 = 0$) computed both with the original uniform penalty are denoted by:

$$J^*_{m,cl} := \sum_{k=0}^{k_{sim}-1} \ell\Big(f(x_k^{cl,m}, \boldsymbol{u}^*(x_k^{cl,m}, m))\Big) \tag{28}$$

$$J^{cstr}_{0,cl} := \sum_{k=0}^{k_{sim}-1} \ell\Big(f(x_k^{cl,0}, \boldsymbol{u}^{cstr}(x_k^{cl,0}))\Big). \tag{29}$$

The comparison between these two closed-loop costs is given in Fig. 3. Obviously, the proposed strategy improves the cost especially for shorter prediction horizons as in this case, the terminal equality constraint leaves no rooms for performance enhancement. This figure also shows that for moderate N, when m increases, the proposed formulation gives similar results as constrained MPC.

Remark 5. The work in Alamir, Bonnay, Bonne, and Trinh (accepted for publication) shows how the framework proposed in the present paper can be applied to the challenging problem of hierarchical MPC control of a cryogenic refrigerator. More precisely, the time varying weighting formulation is applied at the local level

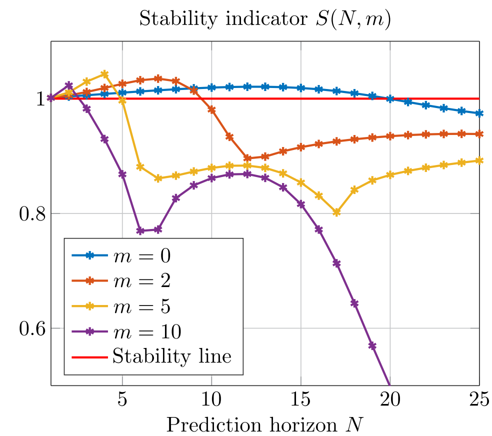

Fig. 1. Evolution of the Stability indicator $S(N, m)$ for different values of the pair (N, m). Note that in the absence of a time varying penalty, a prediction horizon of $N > 20$ would be necessary while $N = 3$ would be sufficient when using time-varying penalty parameter $m = 10$.

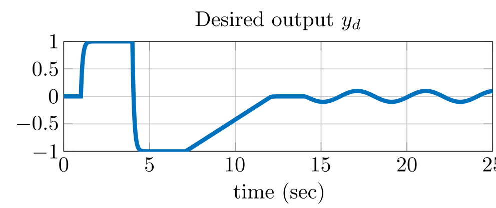

Fig. 2. The desired output profile $y_d(\cdot)$ used in the simulations that produces the results reported in Fig. 3.

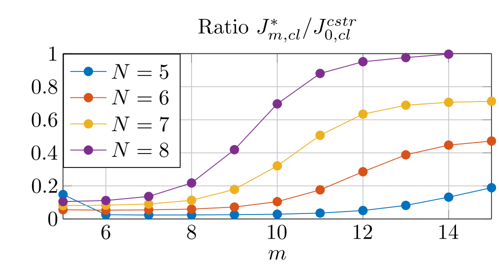

Fig. 3. Performance of the closed-loop systems (expressed in terms of the original uniform-penalty stage cost) for the proposed MPC ($J^*_{m,cl}$) and the traditional MPC with final equality constraints ($J^{xstr}_{0,cl}$).

(subsystems) to solve the individual MPC problems (with respectively 10 and 14 states vectors, 2 control inputs each and 3 coupling signals each). Each individual MPC problem is defined based on the current coupling signal profiles that are sent by the coordinator through a fixed point iteration process. The convergence and the number of needed iterations of this negotiation process as well as the amount of exchanged information highly depend on the length of the prediction horizon and whether the stationarity is achieved at the end of the prediction horizon. The use of the non uniform (in time) weighting as proposed in this theoretical contribution proved crucial in the success of the hierarchical scheme of Alamir et al. (accepted for publication).

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