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Brief paper

Contraction-based nonlinear model predictive control formulation without stability-related terminal constraints☆

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ABSTRACT

Contraction-based nonlinear model predictive control (NMPC) formulations are attractive because they generally require short prediction horizons, and there is no need for the terminal set computation and reinforcement that are common requirements to guarantee stability. However, the inclusion of the contraction constraint in the definition of the underlying optimization problem often leads to non-standard features, such as a need for the multi-step open-loop application of control sequences or the use of multi-step memorization of the contraction level, which may cause unfeasibility in the presence of unexpected disturbances. In this study, we propose a new contraction-based NMPC formulation where no contraction constraint is explicitly involved. The convergence of the resulting closed-loop behavior is proved under mild assumptions. An illustrative example is provided in order to demonstrate the relevance of the proposed formulation.

1. Introduction

Provable closed-loop stability in the majority of nonlinear model predictive control (NMPC) formulations is attributable to the use of terminal constraints on the state. A stringent equality constraint on the state was used in the early formulations (Keerthi & Gilbert, 1988; Mayne & Michalska, 1990). Subsequently, relaxations were introduced by the combined use of terminal set inclusion and an appropriate terminal penalty. The many different ways of choosing these two items were unified by Mayne, Rawlings, Rao, and Scokaert (2000), who showed that the terminal set should be controlled-invariant under some *local* feedback control that makes the terminal penalty a control-Lyapunov function. The terminal set and terminal penalty function are usually computed based on a linear quadratic regulator design if the linearized system around the targeted state is stabilizable; otherwise, methods for computing invariant sets can be used for nonlinear systems (Blanchini, 1999; Limon, Alamo, & Camacho, 2003; Rakovic, Kerrigan, Kouramas, & Mayne, 2005). In particular, a recently proposed scheme (Lazar & Spinu, 2015) can be employed for purely nonlinear systems via extensive use of the finite step Lyapunov function paradigm. Contractive NMPC schemes based on interval analysis (Jaulin, Kieffer, Didrit, & Walter, 2001) have also been proposed (Wan, 2007; Wan, Vehi, & Luo, 2004) and applied to relatively simple two-dimensional robotic examples where this technique is still suitable.

Regardless of the way that the terminal state and terminal penalty pair is computed, the feasibility of the associated terminal constraint generally needs long prediction horizons to be used in the model predictive control (MPC) formulation. Moreover, the presence of this constraint makes the computation of the optimal solution an even more difficult task. This may explain why many practitioners admit to never including these stability-related constraints in their formulations, even in applications where the latter are mainly focused on stabilization.

It was also shown (Alamir & Bornard, 1995) that provable stability can be obtained without a terminal stability-related constraint by using a *sufficiently long* prediction horizon (Grimm, Messina, Tuna, & Teel, 2005; Jadbabaie & Hauser, 2005). More recent results [see Boccia, Grüne, & Worthmann, 2014; Grüne & Pannek, 2011; Grüne, Pannek, Seehafer, & Worthmann, 2010 and the references therein] have provided a deeper analysis regarding this issue. However, the underlying argument remains that with a sufficiently long prediction horizon, the optimal decisions necessarily lead to open-loop trajectories with terminal appropriate properties.

For obvious computational reasons, we might be interested in formulations that involve short prediction horizons and no

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stability-related terminal constraints. Following the previous argumentation, this might appear paradoxical, but it is precisely why contractive formulations may be considered. Indeed, the contraction property for a controlled system is the systematic ability to find a control sequence $\boldsymbol{u}$ that steers the state of the system from its current value x_k to a new state x_{k+N}, where the value of a positive definite function W is contracted by some ratio $\gamma \in (0, 1)$, i.e., $W(x_{k+N}) \leq \gamma W(x_k)$. When $N = 1$, this property is satisfied only for difficult to find standard controlled Lyapunov functions. However, as N becomes slightly higher, the property is true for a wider class of functions, which are referred to as N-step Lyapunov functions (Bobiti & Lazar, 2014). More interestingly, it can be shown (Alamir, 2006; Bobiti & Lazar, 2014) that for stabilizable systems, *any* positive definite function W satisfies the contraction property for an appropriate N. These values of N are generally much smaller than that needed to make standard terminal constraints feasible for a large set of possible initial states.

The difficulty of including the contraction property in the MPC formulation is due to the receding-horizon implementation of the resulting optimal sequence. Indeed, if we assume that an open-loop *contractive* trajectory is found (at instant k) such that $W(x^{ol}_{k+N}) \leq \gamma W(x^{ol}_k)$, then it might still be true that $\gamma W(x^{ol}_{k+1}) > W(x^{ol}_k)$ because W is only a finite-step Lyapunov function and thus not monotonically decreasing. Therefore, if the problem is reformulated at instant $k + 1$ using the constraint $W(x_{k+1+N}) \leq \gamma W(x^{ol}_{k+1})$, then this does not guarantee the closed-loop contraction of W. This explains why two possible alternatives were proposed to enhance closed-loop contraction in the earlier use of the contraction property in the MPC formulation (Kothare, de Oliveira, & Morari, 2000). In the first, the contractive open-loop trajectory is applied in open-loop until contraction occurs. In the second, the contraction level $\gamma W(x_k = x_{past})$ is memorized and employed when formulating the subsequent optimization problems under the constraint that $\min_{i=1,\ldots,N} W(x_{k+i}) \leq \gamma W(x_{past})$ until contraction occurs at some instant $k + i^*$ at which the updating rule $x_{past} = x_{k+i^*}$ is adopted and the process is repeated. These two alternatives are obviously not satisfactory because the system is left in open-loop in the former, and the use of a memorized level might lead to unfeasibility in the presence of disturbance in the second. These drawbacks motivated the contractive scheme proposed by Alamir (2007), which uses no stability-related constraint in the MPC formulation. In the present study, we improve the formulation proposed in Alamir (2007) by using a standard cost function together with a stability-dedicated penalty term, whereas in Alamir (2007), only the contractive function was used in the cost function, thereby making the formulation of Alamir (2007) exclusively focused on stabilization. Moreover, state constraints are considered whereas (Alamir, 2007) only considered control saturation.

The remainder of this paper is organized as follows. First, some definitions and notations are introduced in Section 2, as well as the assumptions needed to derive the main result. Section 3 presents the proposed contractive MPC formulation and the main convergence results. In Section 4, we propose a modified formulation to address some computational issues. Finally, an illustrative example is given in Section 5. Note that due to the lack of space, all of the proofs are omitted and they are reported in the available extended version (Alamir, 2016).

2. Definitions and notation

This study considers nonlinear systems of the form:

$$x_{k+1} = f(x_k, u_k), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the control input. Given a sequence $\boldsymbol{u} := (u^{(1)}, \ldots, u^{(N)}) \in \mathbb{R}^{m\times N}$ of future control inputs together with some initial state x, the resulting state trajectory is denoted by $\boldsymbol{x}^{\boldsymbol{u}}(x) := (x^{(1)}, \ldots, x^{(N)})$, where $x^{(1)} = f(x, u^{(1)})$ and $x^{(i+1)} = f(x^{(i)}, u^{(i+1)})$. In the sequel, the notations $\boldsymbol{u}_\ell = u^{(\ell)}$ and $\boldsymbol{x}^{\boldsymbol{u}}_\ell(x) = x^{(\ell)}$ are used when needed, i.e., $\boldsymbol{x}^{\boldsymbol{u}}_\ell(x)$ is the state reached ℓ steps ahead starting from the initial state x and by applying the sequence of controls $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_\ell$.

Regarding the constraints, it is assumed that u belongs to a compact set $\mathbb{U} \subset \mathbb{R}^m$ and that the set of admissible states is given by $\mathbb{G} := \{x \mid g(x) \leq 0\}$. Moreover, the following assumption is made in the sequel.

Assumption 1. $\mathbb{G}$ is a $\mathbb{U}$-controlled-invariant set that contains a neighborhood of the origin.

Regarding the contraction property, the following assumption is made.

Assumption 2. A positive definite function W, a contraction factor $\gamma \in (0, 1)$, and a prediction horizon N exist such that $\forall x \in \mathbb{G}$, $\exists \boldsymbol{u} \in \mathbb{U}^N$ such that:

$$\forall \ell \in \{1, \ldots, N\} \quad \boldsymbol{x}^{\boldsymbol{u}}_\ell(x) \in \mathbb{G} \tag{2}$$

$$\underline{W}(x, \boldsymbol{u}, N) := \min_{\ell=1}^{N}\Big[W(\boldsymbol{x}^{\boldsymbol{u}}_\ell(x))\Big] \leq \gamma W(x). \tag{3}$$

In the sequel, the argument of the minimization problem in (3) over a prediction horizon of length N is denoted by $\ell_{opt}(x, \boldsymbol{u}, N)$. More generally, given a prediction horizon $q \leq N$, the following notation is used:

$$\ell_{opt}(x, \boldsymbol{u}, q) := \arg \min_{\ell\in\{1,\ldots,q\}} W(\boldsymbol{x}^{\boldsymbol{u}}_\ell(x)). \tag{4}$$

It is assumed that a stage cost function $L(x, u)$ is used to express the control objective. For a control sequence $\boldsymbol{u}$, the following notation is used:

$$\Phi(x, \boldsymbol{u}, q) := \sum_{\ell=1}^{q} L(\boldsymbol{x}^{\boldsymbol{u}}_\ell(x), \boldsymbol{u}_\ell). \tag{5}$$

Moreover, the following assumption is made regarding the behavior of L inside the admissible domain

Assumption 3. $\exists \bar{L} > 0$ such that:

$$\forall (x, u) \in \mathbb{G} \times \mathbb{U}, \qquad 0 \leq L(x, u) \leq \bar{L}. \tag{6}$$

Moreover, $Q(x) := L(x, 0)$ is a positive definite function of the state such that $Q(x) \leq L(x, u)$ for all u.

3. The contractive formulation

For any $z > 0$ and any state $x \in \mathbb{G}$, let us define the following optimization problem denoted by $\mathcal{P}(x, z)$:

$$\min_{(\boldsymbol{u},q)} \left[J^{(x,z)}(\boldsymbol{u}, q)\right] := z \cdot \Phi(x, \boldsymbol{u}, q) + \alpha \underline{W}(x, \boldsymbol{u}, q) \tag{7}$$

$$\text{under } \boldsymbol{x}^{\boldsymbol{u}}_\ell(x) \in \mathbb{G} \quad \forall \ell \in \{1, \ldots, q\} \tag{8}$$

$$\text{and } (\boldsymbol{u}, q) \in \mathbb{U}^N \times \{1, \ldots, N\}, \tag{9}$$

where z is an internal state of the controller with the dynamics defined by (14) in the following, and q is the free-prediction horizon, which is considered to be a decision variable in the proposed formulation. Let us denote $\boldsymbol{u}^*(x, z)$ and $q^*(x, z)$ as the optimal solutions (if any) of the optimization problem (7)–(9). Moreover, the corresponding values of $\underline{W}$, Φ, J, and ℓ_{opt} are denoted by:

$$\underline{W}^*(x, z) := \underline{W}(x, \boldsymbol{u}^*(x, z), q^*(x, z)) \tag{10}$$

$$\Phi^*(x, z) := \Phi(x, \boldsymbol{u}^*(x, z), q^*(x, z)) \tag{11}$$

$$J^*(x,z) := J^{(x,z)}(\boldsymbol{u}^*(x,z), q^*(x,z)) \quad (12)$$
$$\ell^*(x,z) := \ell_{opt}(x, \boldsymbol{u}^*(x,z), q^*(x,z)). \quad (13)$$

The dynamics of the controller's state z are given by:

$$z_{k+1} = h(x_k, z_k) := \begin{cases} z_k & \text{if } W(x_k) > z_k \\ \beta z_k & \text{if } W(x_k) \le z_k, \end{cases} \quad (14)$$

where $\beta \in (0, 1)$ is some fixed constant, which can be viewed as a parameter of the controller. This completely defines the MPC feedback by:

$$z^+ = h(x,z) \quad (15)$$
$$K_{\text{MPC}}(x,z) := \boldsymbol{u}_1^*(x,z). \quad (16)$$

Remark 1. The role of z can be better understood by dividing the cost function by z to obtain:

$$z\Phi + \alpha \underline{W} \quad \rightarrow \quad \Phi + \alpha\left[\frac{\underline{W}}{z}\right],$$

which shows that z serves as a normalization for $\underline{W}$, thereby allowing the definition of the **state-independent condition** on how high α must be to enforce the contraction of W on the closed-loop trajectory. This is because the contraction property is given in multiplicative form.

Now, the fact that z allows normalization comes from the fact that immediately after the switch defined by (14), we have $z = \beta W(x)$. The fact that the cost is multiplied by z simply avoids dividing by 0 because z is expected to converge toward 0.

An interesting consequence of this formulation is that according to (14) and the definition of the cost function, the performance-related cost can be highly weighted in the large while when the desired region is approached, the cost focuses on the stability-related cost W.

In the following, some preliminary results are derived that are used later in the proof of the main result. The first result and its Corollary 1 give an explicit and computable upper bound on the optimal cost.

Lemma 1. ***If*** *Assumptions 2, 3 are satisfied,* ***then*** $\forall(x,z) \in \mathbb{G} \times \mathbb{R}_+$, *$\mathcal{P}(x,z)$ is feasible. Moreover,*

$$J^*(x,z) \le zN\bar{L} + \alpha\gamma W(x) \quad (17)$$

and the minimum value of the contractive map W is obtained at the end of the trajectory, i.e.,

$$\ell^*(x,z) = q^*(x,z). \quad (18)$$

Proof. See Alamir (2016).

Corollary 1. *Under Assumptions 2 and 3,* ***if***

(1) $W(x) > z$;
(2) $\alpha \ge 2N\bar{L}/(1-\gamma)$,

then *the optimal solution is s.t.:*

$$J^*(x,z) \le \left[\frac{1+\gamma}{2}\right]\alpha W(x). \quad (19)$$

Proof. See Alamir (2016).

The two preceding results determine the bounds on the optimal cost, whereas the following two lemmas characterize the behavior of two successive values of the optimal cost at instants k, where $W(x_k)$ is still greater than z_k. Lemma 2 characterizes this behavior in the case where $q^*(x_k, z_k) > 1$, whereas Lemma 3 gives this characterization when $q^*(x_k, z_k) = 1$.

Lemma 2. *Under Assumptions 2 and 3,* ***if***

(1) $q^*(x_k, z_k) > 1$;
(2) $W(x_k) > z_k$,

then *the following inequality holds:*

$$J^*(x_{k+1}, z_{k+1}) \le J^*(x_k, z_k) - z_k Q(x_{k+1}), \quad (20)$$

where $Q(\cdot)$ is the positive definite function invoked in Assumption 3.

Proof. See Alamir (2016).

Lemma 3. *Under Assumptions 2 and 3,* ***if*** *the following conditions hold:*

(1) $q^*(x_k, z_k) = 1$,
(2) $z_{k+1} < W(x_{k+1})$,
(3) $\alpha \ge 2N\bar{L}/(1-\gamma)$,

then *the following inequality holds:*

$$J^*(x_{k+1}, z_{k+1}) \le J^*(x_k, z_k) - z_k Q(x_{k+1}). \quad (21)$$

Proof. See Alamir (2016).

Lemmas 2 and 3 allow us to establish the following corollary.

Corollary 2. *Under Assumptions 2 and 3,* ***if*** *the penalty $\alpha \ge 2N\bar{L}/(1-\gamma)$ is used, then for all initial $z_0 > 0$, the set $\{x = 0\}$ is an accumulation set for the closed-loop dynamic system, i.e., there is a subsequence of $\{x_k\}_{k\ge 0}$ that converges to 0.*

Proof. See Alamir (2016).

Corollary 2 proves that the trajectory of the state regularly visits an always smaller neighborhood of the targeted state $x = 0$ at an increasingly infinite instant of time. We still need to analyze the asymptotic behavior of the excursion of the state trajectory between these instants, where a local assumption is needed regarding the property of the system in an arbitrarily small neighborhood of the origin.

Assumption 4. The origin is **locally** N-step stabilizable. More precisely, there is a local neighborhood $\mathcal{V}$ of the origin such that for all $x_0 \in \mathcal{V}$, there is an admissible control sequence $\boldsymbol{u}$ such that $\boldsymbol{\Pi}_c\left[x_N^{\boldsymbol{u}}(x_0)\right] = 0$ [where $\Pi_c(x)$ is the locally controllable part of x while the uncontrolled part is asymptotically stable]. Moreover, the corresponding state trajectory is entirely contained in $\mathbb{G}$.

Proposition 1. ***Assume that:***

(1) *Assumptions 2–4 are satisfied.*
(2) *The penalty α involved in the cost function (7) is such that*

$$\alpha \ge \frac{2N\bar{L}}{1-\gamma}. \quad (22)$$

Then, *$x = 0$ is asymptotically stable for the closed-loop associated with the MPC law defined by (7)–(9) for all initial states (x,z) such that $x \in \mathbb{G}$ and $z > 0$.*

Proof. See Alamir (2016).

Note that condition (22) in Proposition 1 is a quantified realization of the stabilizing role of the terminal penalty in the context of the absence of the terminal constraint, as suggested by Grüne et al. (2010) (see the discussion in Section 8.2 regarding this issue).

4. Implementation issues

In this section, we show that the formulation of (7)–(9), which involves the integer variable q (representing the free prediction horizon), can be replaced by a new formulation where a predicted sequence $\boldsymbol{u}$ and prediction horizon q can be obtained using a two-stage algorithm, in which each stage involves a fixed prediction horizon ($\leq N$) that is not a decision variable. Moreover, when implemented in a receding-horizon manner, the computed sequence induces the previously established stability. In particular, the two steps are defined as follows.

(1) First, problem (7)–(9) is solved for $z = 0$ and with the additional constraint that $q = N$. More precisely, the following **fixed-horizon** optimization problem is solved:

$$\min_{\boldsymbol{u}\in\mathbb{U}^N}\Big[\underline{W}(x,\boldsymbol{u},N)\Big] \mid \boldsymbol{x}_\ell^{\boldsymbol{u}}(x)\in\mathbb{G} \quad \forall \ell\leq N \tag{23}$$

to obtain the index $\ell_N^\ddagger := \ell^*(x,0)$. Note that this is a standard optimization problem in the continuous variable $\boldsymbol{u}$.

(2) Using the resulting index $\ell_N^\ddagger$, the following fixed-horizon optimization problem is solved where the original stage cost is re-introduced with the same penalty z:

$$\boldsymbol{u}^\ddagger(x,z)\leftarrow \min_{\boldsymbol{u}\in\mathbb{U}^{\ell_N^\ddagger}}\Big[J^{(x,z)}(\boldsymbol{u},\ell_N^\ddagger)\Big] \mid \boldsymbol{x}_\ell^{\boldsymbol{u}}(x)\in\mathbb{G} \quad \forall\ell\leq\ell_N^\ddagger, \tag{24}$$

which is simply (7)–(9), where $q = \ell_N^\ddagger$ is used to eliminate the integer decision variable q. Again, this yields a standard optimization problem in the continuous variable $\boldsymbol{u}$.

Note that both problems inherit the advantages of the contraction-based formulation in the preceding section, i.e., the short prediction horizon and the absence of stability-related terminal constraints. Moreover, we obtain the following convergence result.

Proposition 2. ***If the following conditions hold:***

(1) *Assumptions 2–4 are satisfied.*
(2) *The penalty α involved in the definition of the cost function $J^{(x,z)}$ used in (24) satisfies (22),*

Then, *$x = 0$ is asymptotically stable for the closed-loop associated with the MPC law defined by $\boldsymbol{u}_1^\ddagger(\cdot,\cdot)$ for all initial states (x,z) such that $x\in\mathbb{G}$ and $z>0$.*

Proof. See Alamir (2016).

Remark 2. It might be argued that the proposed formulation is comparable to standard formulations where the terminal constraint on the state is removed and replaced by an additional highly weighted term. Actually, these two formulations differ due to the following reasons at least.

(1) First, even if the final constraint is transformed into a weighted term, it is still necessary to compute the corresponding terminal set, which remains a tedious and dissuasive task, at least for nonlinear systems.
(2) Even if the terminal set is computed, the actual penalty that should affect the corresponding term in order to obtain provable closed-loop stability is still required. In the present formulation, a generic and exact computation of this term is given.
(3) It should be considered that the relevance of the terminal set holds mainly for a long prediction horizon that is sufficient to reach the terminal set. In the proposed formulation, the prediction horizon involved should be much shorter (e.g., see the example in the next section).

5. Illustrative example

Let us consider the discrete-time version of the nonholonomic system given by:

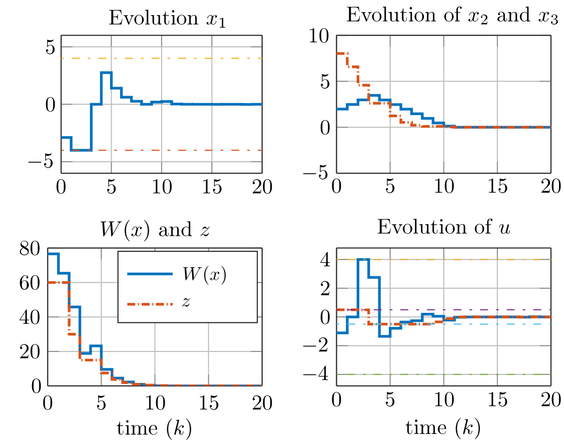

Fig. 1. Evolution of the closed-loop system for $N = 3$ and the stage cost $L_1(x,u)$ given by (28).

$$x_1^+ = x_1 + u_1 \tag{25}$$
$$x_2^+ = x_2 + u_2 \tag{26}$$
$$x_3^+ = x_3 + x_1u_2, \tag{27}$$

where the control u is saturated through $|u_i| \leq \bar{u}_i$, $i = 1,2$, while the state x is constrained by $x_1\in[-\rho,+\rho]$, $|x_i|\leq b$ for $i = 2,3$. If we consider the constraint set given by the constraints map $g(x) = (|x_1|-\rho, x_1^2+x_3^2-b^2)^T$, then the following result holds (see Alamir, 2016).

Proposition 3. *If $\bar{u}_1 \geq 2\rho$ and $\bar{u}_2 = \mu b$, then Assumption 2 holds for any b, $N\geq 3$, and $\gamma\in(0,1)$ that satisfy $\gamma\geq 1-\mu$. Moreover, the contraction map $W(x) = \|x\|_2^2$ can be used.*

In the following, $\bar{u}_1 = 2\rho$ and $\bar{u}_2 = \mu b$ are used. Note that μ is the ratio between the bound $\bar{u}_2$ on u_2 and the radius of the admissible region in (x_2,x_3). The result of Proposition 3 states that this ratio can be as small as desired and that the contraction property still holds for $N = 3$ and $\gamma = 1-\mu$. This obviously shows that although the prediction horizon needed by standard terminal region MPC formulations will increase indefinitely, the contractive-based formulation still only needs $N = 3$ for its requirements to be satisfied. In the simulation, the following two different stage costs are considered:

$$L_1 = \|x\|^2 + 0.1\|u\|^2 \tag{28}$$
$$L_2 = 0.01x_1^2 + x_2^2 + 100(x_2-x_3)^2 + 0.1\|u\|^2, \tag{29}$$

which correspond to the following upper bounds used in the computation of the convenient terminal penalty α involved in Propositions 1 and 2:

$$\bar{L}_1 := \rho^2 + 2b^2 + 0.1\left[4\rho^2 + (\mu b)^2\right] \tag{30}$$
$$\bar{L}_2 := 0.01\rho^2 + 401b^2 + 0.1\left[4\rho^2 + (\mu b)^2\right]. \tag{31}$$

Depending on the stage cost used, α is taken as equal to the minimal value required by the inequality (22) of Proposition 1. In the simulations, the following values are used $\mu = 0.05$, $b = 10$, $\beta = 0.5$, and $\rho = 4$. Fig. 1 shows the closed-loop behavior when $N = 3$ is used, where the non-monotonic decrease in the penalty function W can be clearly observed. The fact that the stage cost influences the behavior of the closed-loop system can be observed by comparing Figs. 2 and 3, where the stage costs L_2 and L_1 are used successively. Indeed, when L_2 is used, the difference $|x_2 - x_3|$ is reduced compared with the case when L_1 is used.

6. Conclusion

In this study, we proposed a new contractive NMPC formulation to accommodate short prediction horizons, which avoids the

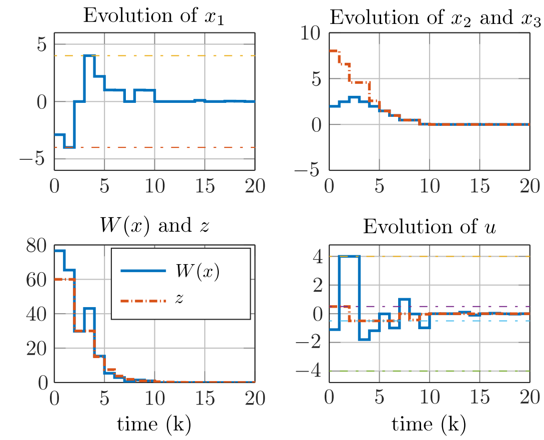

Fig. 2. Evolution of the closed-loop system for $N = 5$ and the stage cost $L_2(x, u)$ given by (28).

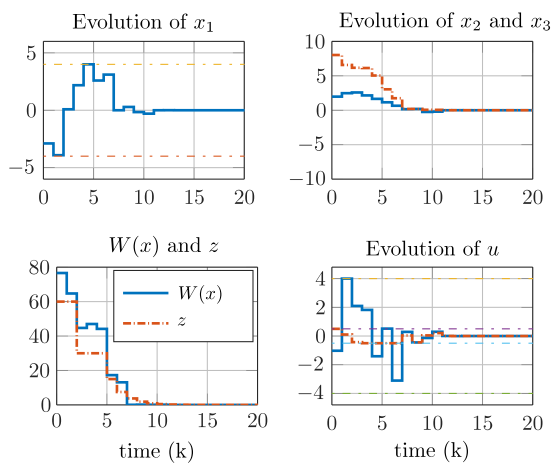

Fig. 3. Evolution of the closed-loop system for $N = 5$ and the stage cost $L_1(x, u)$ given by (28).

use of stability-related terminal state constraints. We derived an efficient two-step procedure to solve the underlying optimization problem. Our ongoing investigations aim to identify a realistic and relevant control problem that could be addressed by the proposed framework. For such systems, it is likely that all the assumptions of the proposed formulation cannot be proved rigorously. However, the use of probabilistic certification can lead to applicable design procedures.

References

Alamir, M. (2006). *Lecture Notes in Control and Identification Sciences: vol. 339. Stabilization of nonlinear systems using receding-horizon control schemes: a parametrized approach for fast systems*. Springer, ISBN: 1-84628-470-8.

Alamir, M. (2007). A low dimensional contractive NMPC scheme for nonlinear systems stabilization: Theoretical framework and numerical investigation on relatively fast systems. In *Assessment and future directions of nonlinear model predictive control* (pp. 523–535). Springer.

Alamir, M. (2016). A new contraction-based NMPC formulation without stability-related terminal constraints. arXiv:1602.00547.

Alamir, M., & Bornard, G. (1995). Stability of a truncated infinite constrained receding horizon scheme: the general discrete nonlinear case. *Automatica*, *31*(9), 1353–1356.

Blanchini, F. (1999). Set invariance in control. *Automatica*, *35*(11), 1747–1767.

Bobiti, R., & Lazar, M. (2014). On the computation of Lyapunov functions for discrete-time nonlinear systems. In *System theory, control and computing (ICSTCC), 2014 18th international conference*. Oct (pp. 93–98).

Boccia, A., Grüne, L., & Worthmann, K. (2014). Stability and feasibility of state constrained {MPC} without stabilizing terminal constraints. *Systems & Control Letters*, *72*, 14–21.

Grimm, G., Messina, M. J., Tuna, S. E., & Teel, A. R. (2005). Model predictive control: for want of a local control Lyapunov function, all is not lost. *IEEE Transactions on Automatic Control*, *50*(5), 546–558.

Grüne, L., & Pannek, J. (2011). *Nonlinear model predictive control. Theory and algorithms*. Springer-Verlag.

Grüne, L., Pannek, J., Seehafer, M., & Worthmann, K. (2010). Analysis of unconstrained nonlinear MPC schemes with time-varying control horizon. *SIAM Journal on Control and Optimization*, *48*(8), 4938–4962.

Jadbabaie, A., & Hauser, J. (2005). On the stability of receding horizon control with a general terminal cost. *IEEE Transactions on Automatic Control*, *50*(5), 674–678.

Jaulin, L., Kieffer, M., Didrit, O., & Walter, E. (2001). *Applied interval analysis*. Springer.

Keerthi, S. S., & Gilbert, E. G. (1988). Optimal infinite horizon feedback laws for a general class of constrained discrete-time systems: Stability and moving horizon approximations. *Journal of Optimization Theory and Applications*, *57*, 265–293.

Kothare, S., de Oliveira, L., & Morari, M. (2000). Contractive model predictive control for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, *45*.

Lazar, M., & Spinu, V. (2015). Finite-step terminal ingredients for stabilizing model predictive control. *IFAC-PapersOnLine*, *48*(23), 9–15. 5th IFAC Conference on Nonlinear Model Predictive Control NMPC 2015 Seville, Spain, 1720 September 2015.

Limon, D., Alamo, T., & Camacho, E.F. (2003). Robust MPC control based on a contractive sequence of sets. In *Decision and Control, 2003. Proceedings. 42nd IEEE conference on. Vol. 4* (pp. 3706–3711).

Mayne, D. Q., & Michalska, H. (1990). Receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, *35*, 814–824.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., & Scokaert, P. O. M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, *36*, 789–814.

Rakovic, S. V., Kerrigan, E. C., Kouramas, K. I., & Mayne, D. Q. (2005). Invariant approximations of the minimal robust positively invariant set. *IEEE Transactions on Automatic Control*, *50*(3), 406–410.

Wan, J. (2007). *Computationally reliable approaches of contractive MPC for discrete-time systems*. (Ph.D. thesis), University of Girona.

Wan, J., Vehi, J., & Luo, N. (2004). Nonlinear model predictive control via interval analysis. In *Proceedings of the International Symposium on Nonlinear Control Systems (Nolcos)* (pp. 949–952).

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