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Estimation of energy saving thanks to a reduced-model-based approach: Example of bread baking by jet impingement

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ABSTRACT

In this paper, a reduced order mechanistic model is proposed for the evolution of temperature and humidity during French bread baking. The model parameters are identified using experimental data. The resulting model is then used to estimate the potential energy saving that can be obtained using jet impingement technology when used to increase the heat transfer efficiency. Results show up to 16% potential energy saving under certain assumptions.

1. Introduction

Food industry represents a non-negligible part of the energy consumption in industrial countries [1], and the improvement of energy efficiency through combinations of energy sources can be a relevant approach to reduce the energy cost [2]. In this paper, the authors focus on the bakery sector, where baking is a particularly energy expensive step.

Baking is the final stage of the bread making process during which, the control of the exposition to high temperature and the resulting humidity is essential in obtaining the desired bread characteristics. An exhaustive mastering of all the processes that take place during this final stage is beyond hope despite of many works that have been dedicated to a better understanding of the underlying physico–chemical phenomena [3–5].

As far as modelling of complex processes is concerned, there are two different points of view:

The knowledge-based point of view in which the modelling effort seeks a better understanding of the underlying phenomena. This understanding makes it possible to predict the evolution of the dynamic process under a wide range of operating scenarios. Such models generally involve a huge number of physical parameters. These parameters are quite difficult to estimate due to the strong coupling of the involved phenomena and the time-varying nature of these parameters that may also depend on the process history. Concerning baking, there has been an increasing number of studies developing heat and mass transfer models [6–8], or coupling fluid dynamics and heat/mass transfer [9,10].

The identification point of view in which one tries to obtain a mechanistically faithful reproduction of the process behaviour as far as a restricted family of scenarios is involved. The model is then obtained by identification (optimization based) procedure based on the sole measured input/output data. The validity of the so obtained model is then clearly restricted to the above family of scenarios and the extrapolation power of the resulting model is far lower than the one shown by knowledge-based models. On the other hand, a rather small number of parameters are involved that are not always physically meaningful. Fortunately though, since these parameters are specially introduced to reproduce the process behaviour, they can be determined based on decoupled optimization steps. Such approach has been applied to bakery process, for data analysis and classification by Rousu [11] and for product design and optimization by Hadiyanto [12].

Whether to use knowledge-based or mechanistic identified model is goal-dependant. If understanding of phenomena is the main goal, then only the knowledge-based approach is to be used regardless the inherent difficulties. However, if one aims at

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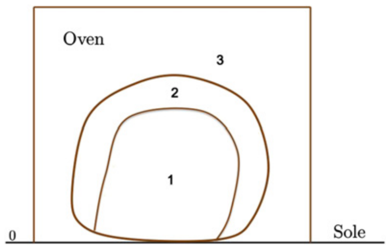

Fig. 1. Vertical section of the French bread dough showing the decomposition into adjacent regions.

optimizing a given process by introducing slight modifications of its operational parameters, then the second approach may be advantageous due to the fact that it is more easily achievable and is still valid over the operational domain of interest.

The work reported in this paper is a part of a national French research grant[1] that aims at investigating energy saving new technologies in French bakery. (This stake is all more significant as bread is produced in a few tens thousands craft production units scattered over the French territory). As far as energy saving is targeted and that attention is focused on the French bread, the second approach may be relevant. Indeed, the initial conditions are rather imposed and the family of temperature profiles cannot be completely arbitrary. Therefore, it can be conjectured that a faithful mechanistic model that reproduces the main characteristic profiles (temperature and water content on the surface and inside the bread during the baking process) can be correctly used in the above mentioned optimization process.

The paper is organized as follows: the derivation of the mechanistic model is given in Section 2. The calibration and the validation of the proposed model are shown in Section 3 using experimental results. As the optimization step needs the thermal characteristic of the oven to be known, such a characterization is given in Section 4 using dedicated experiments. The formulation and a first solution of the energy saving using jet impingement technique are investigated in Section 5 which begins by a presentation of the principle of jet impingement and the oven that is under construction within the project mentioned above.

2. Derivation of the mechanistic model

In this section a step-by-step presentation of the mechanistic model is proposed. First, the notation used throughout this section is stated, and then the different modules of the proposed model are introduced.

2.1. Definitions and notation

The modelling approach is based on the decomposition of the dough into two regions (the dough or crumb (1), and the crust (2)) (Fig. 1). The indices 0 and 3 refer to the sole and the oven respectively.

Related to Fig. 1, the following notation is used:

- T_i is the mean temperature (K) in the i-th region
- m_i^l is the liquid water mass (kg) in the i-th region
- m_i^v is the water vapour mass (kg) in the i-th region
- $I_i$is the set of indices of regions that are adjacent to the i-th region, namely:

$$I_1 = \{0, 2\} \quad , \quad I_2 = \{0, 1, 3\} \tag{1}$$

[1] ANR ALIA 2008 – BRAISE (Boulangerie RAISonnée et Efficacité énergétique).

2.2. Heat exchange model

The mechanisms of heat exchanges between the different regions (sole/crust, oven/crust, crust/crumb) can be resumed in a mechanistical equation representing the evolution of the temperature T_i:

$$\frac{dT_i}{dt} = \alpha(\xi)\cdot\left[\beta\cdot\sum_{j\in I_i} S_{ij}(T_j - T_i) - Q_i^{\text{vap}} - Q_i^r\right] \tag{2}$$

in which ξ is a dimensionless scalar positive parameter that summarizes the heating history of the process according to:

$$\dot{\xi} = \beta_0\cdot T_3 \tag{3}$$

where β_0 is chosen such that ξ equals 1 at the end of the baking process.$\alpha(\xi)$ is a polynomial in ξ that is taken to be of the form:

$$\alpha(\xi) = \alpha_0 + \alpha_1\cdot\xi \tag{4}$$

which enables a smooth take-off like behaviour to be represented by choosing the α_i's coefficients conveniently. Note that the use of the factor $\alpha(\xi)$ expresses the fact that due to the internal transformations that take place inside the dough, the efficiency of the heat transfer is most likely to be *time*-varying. However, by time, one must read *time during which the dough is immersed in high temperature*. This explains the use of the process time ξ which is *steered* by the oven temperature T_3 rather than the physical time t.

Note that the first term in the brackets stands for the heat exchange between adjacent regions while Q_i^{vap} represents the heat used in the evaporation process. This term is more precisely described in the sequel.

Finally, the term Q_i^r represents the heat used in the different reactions that take place inside the dough during baking such as starch gelatinization and gluten aggregation.

$$Q_i^r = \mu_r\cdot\left[1 + \tanh\left(\beta_r\frac{T_i - T_r}{T_r}\right)\right] \tag{5}$$

which clearly refers to reaction that needs some minimal temperature to take place since for small values of T_i, the term is quite small.

2.3. The evaporation model

The evaporation term Q_i^{vap} is taken to be proportional to the lack of gas-phase water when compared to a liquid–vapour equilibrium value. Namely:

$$Q_i^{\text{vap}} = \mu_v\cdot c\cdot\left[\Phi_{\text{eq}}\left(T_i, m_i^l, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] \tag{6}$$

where c is the water heat capacity (kg/J), $\Phi_{\text{eq}}(T_i, m_i^l, d_1^{(i)}, d_2^{(i)})$ represents the value of the steady quantity of water vapour in region i in case the temperature T_i and the liquid water mass m_i^l are maintained constant. The parameters $d_1^{(i)}$ and $d_2^{(i)}$ are used in the mechanistic definition of Φ_{eq} according to:

$$\Phi_{\text{eq}}\left(T_i, m_i^l, d_1^{(i)}, d_2^{(i)}\right) = \frac{M}{R}\cdot f(T_i)\cdot\Psi\left(m_i^l, d_1^{(i)}, d_2^{(i)}\right) \tag{7}$$

where:

- $M = 0.018$ kg/mol is the molar mass of the water
- $R = 8.31$ J/K/mol is the perfect gas constant

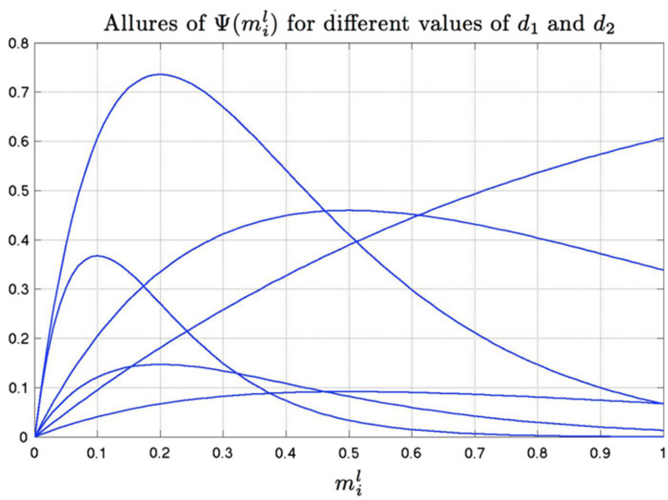

Fig. 2. Allures of the function $\Psi(\cdot, d_1, d_2)$ involved in the definition the water two-phase equilibrium (7) during the baking process. This wide family of behaviours enables to account for the extremely difficult modelling of the various phenomena that take place in the dough during the baking process.

- $f(T)$ is the ratio between the equilibrium pressure $P_{eq}(T)$ (in Pa) and T (in K), namely:

$$f(T) = \frac{P_{eq}(T)}{T} = \frac{1}{T}\exp\left(a - \frac{b}{T} - c \cdot \ln(T)\right) \tag{8}$$

where $a = 38.35$, $b = 5653.7$ and $c = 1.97$.

The function $\Psi(\cdot, d_1, d_2)$ involved in eq. (7) meets the following conditions:

- $\Psi(0, d_1, d_2) = 0$ since equilibrium needs liquid water to be present.
- $\Psi(\cdot, d_1, d_2) \geq 0$.

The presence of internal pressure due to the presence of different reaction induced gases makes it difficult to derive a knowledge-based model that describes the way this equilibrium changes during the baking process. That is the reason why a wide family of behaviours needs to be eligible by the structure of the function. The latter is therefore defined by:

$$\Psi\left(m_i^l, d_1, d_2\right) = d_1 \cdot m_i^l \cdot \exp\left(-d_2 \cdot m_i^l\right) \tag{9}$$

where $d_1 > 0$ and $d_2 > 0$ are positive parameters to be identified from experimental data.

Fig. 2 shows the behaviours of $\Psi(\cdot, d_1, d_2)$ for different values of d_1 and d_2.

Table 1
Experimental results of a set of standard French baking experiments [11]. These data are used to calibrate the mechanistic model proposed in Section 2.

Time (min)	Water mass ratio (%)		T-crust (°C)
	(Dough)	(Crust)	
6	50.2	34.2	103
10	49.3	18.2	120
12.5	49.9	13.2	138
15	50.9	30	140
17.5	50.8	5.3	147
20	49.8	9.0	155
25	49.9	6.1	160
30	50.3	3.0	162
35	49.5	3.7	163
40	49.6	3.3	164

Table 2
The values of the tuned model parameter in order to match the experimental data given in Table 1.

Parameter	Value	Unit	Equation
β_0	1.6×10^{-6}	$K^{-1}\ s^{-1}$	(15)
(d_1^1, d_1^2)	(0.04, 0.9)	($m^3\ kg^{-1}$, kg^{-1})	(16)–(18)
(d_2^1, d_2^2)	(0.06, 0.06)	($m^3\ kg^{-1}$, kg^{-1})	(16)–(18)
(σ_1, σ_2)	(0.001, 3)	($kg\ s^{-1}$, –)	(12)–(18)
(μ_1, μ_2)	(0.02, 3)	($kg\ s^{-1}$, –)	(13)–(18)
μ_v	1.2	$J\ kg^{-1}$	(16)–(18)
μ_r	0.06	J	(16)
T_r	373.15	K	(16)
(α_1, α_2)	(0.6, 0.6)	$K\ J^{-1}\ s^{-1}$	(15) and (16)
β	0.0027	$J\ kg^{-1}$	(15)

2.4. The water mass balance

The evaporation term Q_i^{vap} can then be used to derive the dynamic behaviour of the key variables m_i^l and m_i^v representing the liquid water mass and the water vapour mass respectively, namely:

$$\dot{m}_i^l = -Q_i^{vap} \big/ c \tag{10}$$

$$\dot{m}_i^v = -\dot{m}_i^l - \sum_{j \in I_i} Q_{i \to j}^{mv} \tag{11}$$

where $Q_{i \to j}^{mv}$ stands for the vapour flow rate that leaves region i and enters region j. The mechanistic modelling of this term is based on the following experimental facts:

1- This term must vanish when m_i^v vanishes.
2- The stream of gas moves from the centre to the boundary.
3- When the water content in the crust is beyond some threshold (the crust is dry), the water vapour is bounded to the dough and can no more escape to the oven atmosphere.

To take into account the above fact, the following expression is used:

$$Q_{1 \to 2}^{mv} = \Psi(\xi, \sigma_1, \sigma_2) \times m_1^v \times \left[1 + \tanh\left(\eta\left(\frac{m_2}{m_2^0} - w_c\right)\right)\right] \tag{12}$$

$$Q_{2 \to 3}^{mv} = \Psi(\xi, \gamma_1, \gamma_2) \times m_2^v \tag{13}$$

$$Q_{2 \to 1}^{mv} = Q_{3 \to i}^{mv} = Q_{i \to 0}^{mv} = 0 \tag{14}$$

Indeed, fact 1 is accounted for by using m_i^v in the right hand side of the eqs. (12) and (13). Fact 2 is reproduced through eq. (14). Finally, the last fact is introduced through the term

$$1 + \tanh\left(\eta\left(\frac{m_2}{m_2^0} - w_c\right)\right)$$

which clearly vanishes when m_2 becomes too small when compared to the threshold w_c. The latter may be viewed as the water mass ratio in the dough below which the gas is bound to the dough components. The use of different expressions in eqs. (12) and (13) is justified by the fact that transfer takes place between two regions in the bread or between the bread and the oven atmosphere. Therefore, the process should be expected to follow different behaviours. This implies a priori different parameters (σ_1, σ_2) and (γ_1, γ_2). Moreover, the saturation term (associated to the hyperbolic tangent) is not relevant in eq. (13).

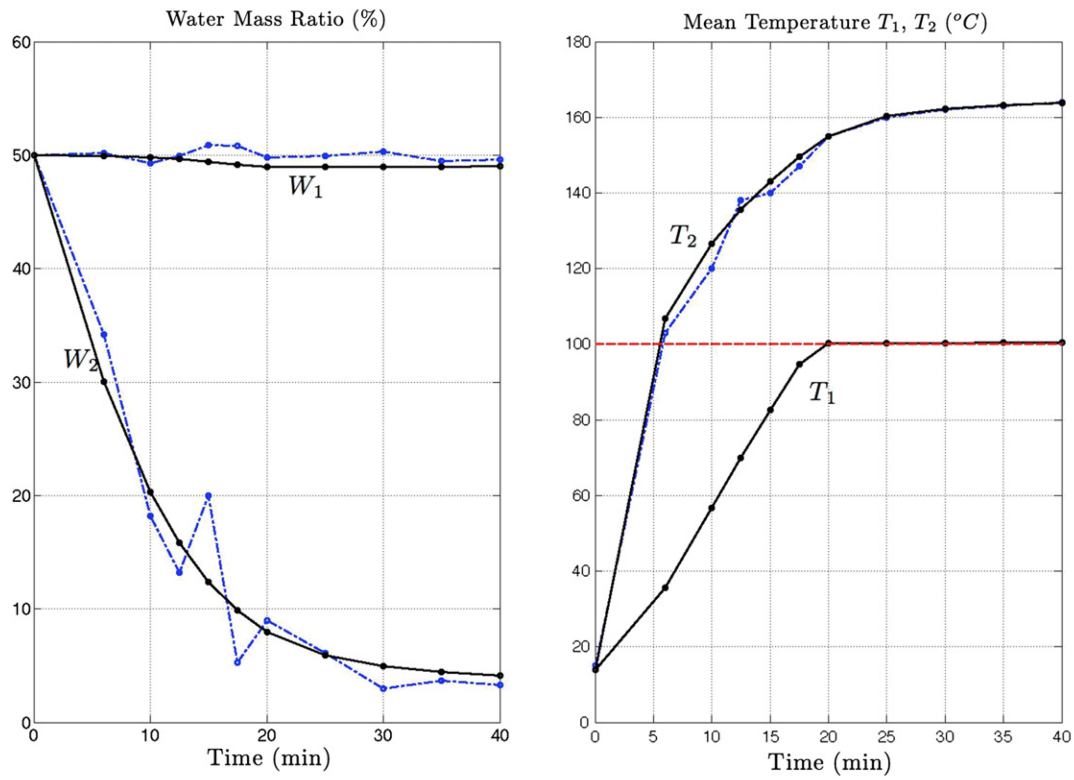

Fig. 3. Comparison between the experimental data (blue-dotted) and the simulated model (black-solid) using the model parameter values given in Table 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2.5. Summary of the mechanistic model

Putting together all the equations presented so far, one obtains the following mechanistic model for the French bread baking ($i \in \{1, 2\}$):

$$\dot{\xi} = \beta_0 \cdot T_3 \tag{15}$$

$$\dot{T}_i = (\alpha_0 + \alpha_1 \xi)\left[\beta \sum_{i \in I_i} S_{ij}(T_j - T_i) - \mu_v\left[\Phi_{eq}\left(T_i, m_i^l, m_i^v, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] - \mu_r\left[1 + \tanh\left(\beta_r \frac{T_i - T_r}{T_r}\right)\right]\right] \tag{16}$$

$$\dot{m}_i^l = \mu_v\left[\Phi_{eq}\left(T_i, m_i^l, m_i^v, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] \tag{17}$$

$$\dot{m}_i^v = -\mu_v\left[\Phi_{eq}\left(T_i, m_i^l, m_i^v, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] - \sum_{j \in I_i} Q_{i \to j}^{mv} \tag{18}$$

where $Q_{i \to j}^{mv}$ is given by eqs. (12) and (13).

In the following section, the calibration and the validation of the mechanistic model proposed above are discussed.

3. Model calibration and validation

The parameters of the mechanistic model have been tuned in order to match the data issued from a standard French baking experiment, where the development of the crust has also been

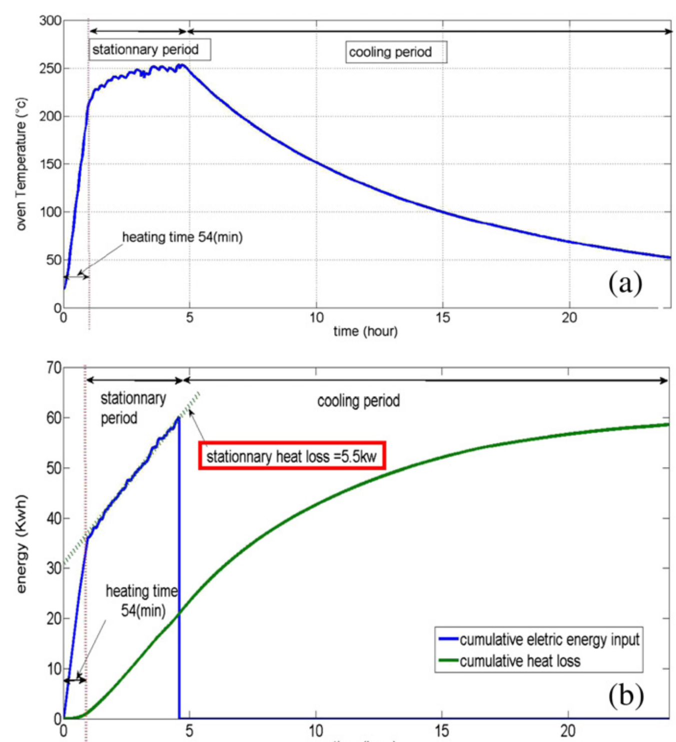

Fig. 4. Experimental results obtained using a BONGARD standard oven commonly used in the French bakery: (a) a scenario containing heating with maximum power phase, followed by a stabilization step and ending with free-cooling phase. (b) Experiment enabling to compute the power needed to maintain a stationary temperature of 250 °C.

confirmed by image analysis follow-up, as described in detail in Ref. [13]. These data are summarized in Table 1 where the evolutions of the following key quantities are given:

- The water mass ratio in the dough–crumb
- The water mass ratio in the crust
- The temperature in the crust.

As for the temperatures in the internal regions of the dough, it is well known that they rise quite rapidly and saturate at temperature that are close to 100 °C.

Optimization has been done using the Matlab® non-convex optimization subroutine “fmincon”. The values of the parameters so obtained are given in Table 2 together with a reminder of the equations in which each parameter is involved.

Fig. 3 shows the comparison between the experimental data (Table 1) and the simulated variables based on the model in which the tuned parameters given on Table 2 are used. The irregular variation in the experimental data of water mass, observed at instants 15 min and 20 min, may be due either to experimental artefact when removing sample or to vapour injection, that is commonly used during French baking. This latter phenomenon is not introduced in the model although this can be easily done.

In the remainder of this paper, a part of the model is used to investigate energy saving when using jet impingement in order to increase the heat transfer between the oven atmosphere and the crust. However, in order to be able to do so, one needs to have some data describing the thermal inertia of the oven as well as the heat efficiency of the actuator. This is done in the next section.

4. Thermal characterization of a standard oven

In order to identify the thermal characteristics of a standard French baking oven, measurements campaign have been conducted in 2010 by the authors during the project BRAISE (see Acknowledgement for more details) using BONGARD® oven, that are widely used in France and Europe. The results of these measurements are shown in Fig. 4. This figure shows two plots: the first one (Fig. 4(a)) shows the evolution of the oven temperature during a scenario containing a heating phase, followed by a stabilization phase and ending with free-cooling phase. This enables the thermal inertia of the oven to be determined.

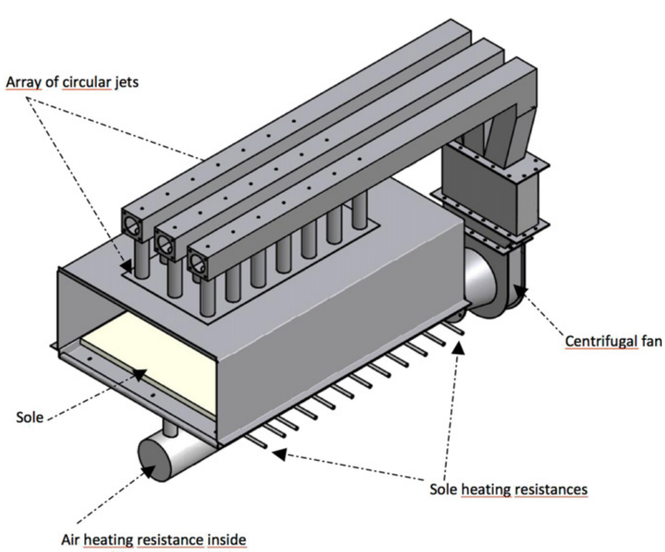

Fig. 5. Sketch of the baking enclosure of the air impingement oven under construction.

The second one (Fig. 4(b)) represents the energy used in maintaining the temperature of the oven at 250 °C. This enables to compute the gain between the normalized heating actuator value and the corresponding consumed heating power.

Based on the above experimental data, the following model of the temperature T_3 can be identified:

$$\dot{T}_3 = \gamma_h \cdot u_1 - \gamma_l \cdot T_3 \tag{19}$$

$$P_h = C_1 \cdot u_1 \tag{20}$$

where

- $u_1 \in [0,\ 1]$ is the heat control level
- γ_h and γ_l are constant coefficients to be identified
- P_h is the consumed heating power corresponding to the actuator value u_1.

First of all, γ_l is computed based on the data contained in Fig. 4(a). More precisely, observing the behaviour of the system between $t = 5$ h and $t = 15$ h clearly shows how the temperature decreases exponentially from 250 °C to 100 °C while $u_1 = 0$ is maintained. This period enables γ_l to be estimated according to:

$$\gamma_l \approx \frac{-1}{\Delta} \ln\left(\frac{T(\Delta)}{T(0)}\right) = \frac{-1}{10 \times 3600} \ln\left(\frac{100+273}{250+273}\right) \approx 9.4 \times 10^{-6} \left(\mathrm{s}^{-1}\right) \tag{21}$$

As for the estimation of γ_h, it can be obtained using the first part of the scenario depicted in Fig. 4(a). Indeed, one can notice that during this period where $u_1 = 1$ is applied, the temperature of the oven raises from 25 °C to 210 °C during 54 min which enables to write:

$$\gamma_h \approx \frac{\gamma_l \cdot \left(T_3(\Delta) - e^{-\gamma_l \cdot \Delta} T_3(0)\right)}{1 - e^{-\gamma_l \cdot \Delta}} \tag{22}$$

$$= \frac{\gamma_l \cdot (210 + 273 - e^{-\gamma_l \cdot (54 \times 60)}(25 + 273))}{1 - e^{-\gamma_l \cdot (54 \times 60)}} = 6.1 \times 10^{-2} \ \left(\mathrm{kg\ s}^{-1}\right) \tag{23}$$

Finally, the gain between the value of u_1 and the corresponding power can be computed based on the power that is necessary to maintain the temperature $T_3 = 250$ °C. This power is experimentally identified to be equal to 5.5 kW (see Fig. 4(b)) which enables to write:

$$P_{\mathrm{stat}}(T_3) = \frac{\gamma_l \cdot C_1 \cdot T_3}{\gamma_h}$$

And hence:

$$C_1 \approx \frac{\gamma_h \times 5500}{\gamma_l \times (250 + 273)} \approx 6.8 \times 10^4 \ \mathrm{W}$$

which completely determines the parameters of the models (19) and (20).

In the following section, the models developed so far are used in order to have a first estimation of the energy saving that can be potentially reached when using the jet impingement technique as a means to increase the heat transfer efficiency between the oven atmosphere and the dough (crust).

5. First investigation of energy saving by jet impingement

In this section, the extended dynamic model in which the jet impingement effect is introduced and the optimal control problem

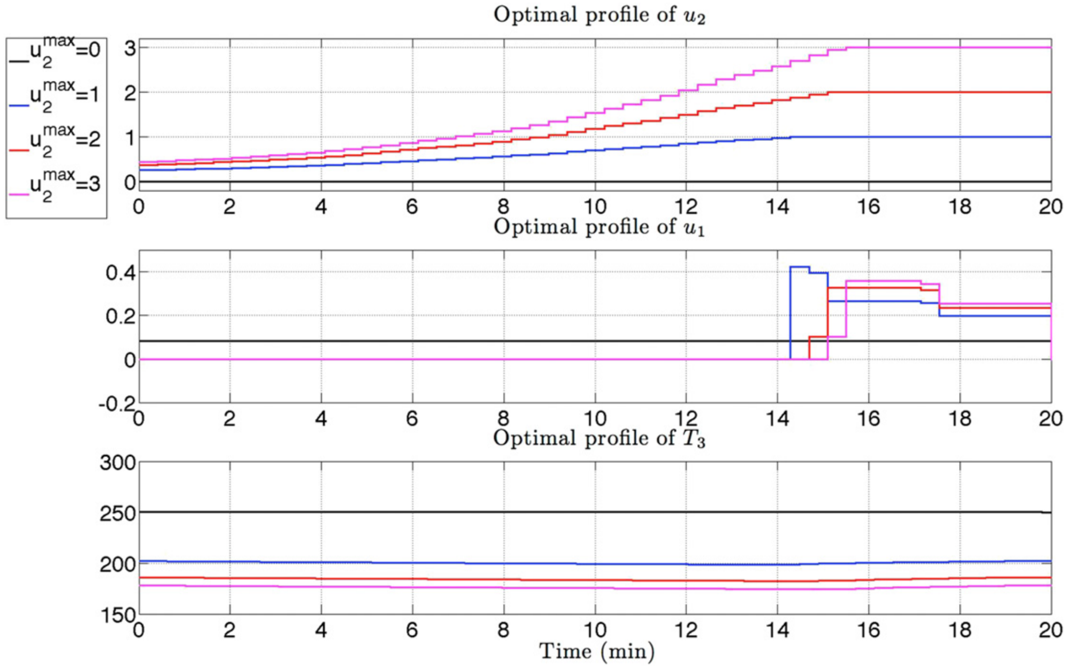

Fig. 6. Computation of the optimal control profile of the heating set-point $u_1(.)$ and the increased heat efficiency transfer coefficient $u_2(.)$ for different values of the maximum allowable efficiency increase coefficient $u_2^{\max}$.

is then stated and solved. However, in order to understand the role of the jet impingement technique and how it can be incorporated in the model, some recalls are proposed and the oven that is under development within our project is briefly described.

5.1. The jet impingement technique

The boundary layer occurring at sole and bread surfaces constitutes a great resistance to convective heat transfer between hot air and the bread surface. Impinging jet can then be used to reduce the thickness of the boundary layer and thus increases the convection [14]. In this study, an impingement jet oven has been developed to study the energy consumption and the heat and mass transfer during bread baking. Its geometrical characteristics were chosen as a compromise between technical considerations and literature reviews [15]. A sketch of the cooking enclosure and the air heating system is presented in Fig. 5. According to the correlations available in the literature, the average Nusselt number due to impingement on a flat plate could represent 5–10 times the Nusselt number due to natural convection [16–18]. It is more difficult to evaluate the gain due to impingement on bread because its volume changes during baking. However the expected gain should be of the same order of magnitude as the one obtained for the sole.

5.2. The extended dynamic model for control

Based on the discussion of the preceding section, the use of jet impingement can be viewed as an additional control ($u_2 \in [0, u_{\max}]$) that increases the coefficient S_2^3 (see eqs. (2) and (16)) which reflects the heat transfer efficiency between the crust (region 2) and the oven's atmosphere. This can be taken into account in the dynamic model by replacing in eq. (16) the term S_{ij} by the term $(1 + \delta_{3,j} \cdot u_2) \cdot S_{ij}$ where $\delta_{3,j}$ is the Kronecker notation ($\delta_{3,j} = 1$ if $j = 3$ and 0 otherwise). Using this notation and putting together the baking model (15)–(18) with the oven model (19) and (20) leads to the following dynamic model with 2 control inputs u_1 and u_2:

$$\dot{T}_3 = \gamma_h \cdot u_1 - \gamma_l \cdot T_3 \tag{24}$$

$$\dot{\xi} = \beta_0 \cdot T_3 \tag{25}$$

$$\begin{aligned}\dot{T}_i = (\alpha_0 + \alpha_1 \cdot \xi)\Bigg[\beta \sum_{j \in I_i} (1 + \delta_{3,j} \cdot u_2) \cdot S_{ij}(T_j - T_i) \\ -\mu_v\left[\Phi_{\text{eq}}\left(T_i, m_i^l, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right]\Bigg] - \ldots \\ -(\alpha_0 + \alpha_1 \cdot \xi) \cdot \mu_r \cdot \left[1 + \tanh\left(\beta_r \frac{T_i - T_r}{T_r}\right)\right]\end{aligned} \tag{26}$$

$$\dot{m}_i^l = -\mu_v\left[\Phi_{\text{eq}}\left(T_i, m_i^l, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] \tag{27}$$

$$\dot{m}_i^v = \mu_v\left[\Phi_{\text{eq}}\left(T_i, m_i^l, d_1^{(i)}, d_2^{(i)}\right) - m_i^v\right] - \sum_{j \in I_i} Q_{i \to j}^{\text{mv}} \tag{28}$$

which is a state space nonlinear dynamic model with 8 state variables and 2 control variables, namely, the heating actuator set-point $u_1 \in [0, 1]$ and the heat transfer improving coefficient $u_2 \in [0, u_2^{\max}]$.

5.3. Optimal control problem formulation

The formulation of the optimal control problem considered in the sequel is based on the assumption according to which:

All the phenomena that take place during the baking process inside the dough are driven by the temperature profile T_2 (shown in Fig. 3 in the case of a standard baking protocol).

Based on this assumption, if one aims at producing an equivalent product using a different heating process that consumes less energy (in particular, a process that involves the degree of freedom u_2), a sufficient condition would be to produce a temperature profile $T_2(\cdot)$

identical to the one obtained using the standard baking protocol. Indeed, this condition ensures in particular that the evolution of the water mass ratio in region 2 will follow the curve drawn in Fig. 3(a). Denoting the targeted temperature profile by $T_2^r(\cdot)$, i.e. the blue-dotted curve in Fig. 3(b) and recalling that the protocol used in this experiment corresponded to a constant oven temperature $T_3 \equiv 250\,^{\circ}\mathrm{C}$, the above target would be reached if the following equality is satisfied:

$$\forall t \in \left[0, t_f\right] \quad ; \quad (1+u_2)\left[T_3(t) - T_2^r(t)\right] = T_3^r(t) - T_2^r(t) \tag{29}$$

where t_f is the baking process duration. Indeed, this would guarantee that the evolution of T_2 exactly tracks T_2^r which (by virtue of the assumption mentioned above) would guarantee that all the key variables would follow their corresponding reference profiles.

Based on the above discussion, the following optimal control problem can be formulated:

Definition 1. (*Problem 1*) *Consider the dynamic model given by*

$$\dot{T}_3 = -\gamma_l \cdot T_3 + \gamma_h \cdot u_1 \tag{30}$$

And let be given:
Two reference profiles $T_2^r(\cdot)$ and $T_3^r(\cdot)$ defined over $[0, t_f]$
A maximum efficiency coefficient $u_2^{\max}$
A coefficient $C_2 = \mu C_1$ that defines the unitary energy cost of the use of u_2
Compute the profiles $u_1^*(\cdot)$ and $u_2^*(\cdot)$ as well as the initial values $T_3(0)$ of the oven temperature that minimizes the energy related cost function defined by:

$$J^* = \min_{u_1(\cdot),\, u_2(\cdot),\, T_3(0)} \int_0^{t_f} [u_1(t) + \mu \cdot u_2(t)]\mathrm{d}t \tag{31}$$

while meeting the following constraints:

$$\forall t \in \left[0, t_f\right] \quad ; \quad (1+u_2(t))\left(T_3(t) - T_2^r(t)\right) = T_3^r(t) - T_2^r(t) \tag{32}$$

$$\forall t \in \left[0, t_f\right] \quad ; \quad u_2(t) \in \left[0, u_2^{\max}\right] \tag{33}$$

$$\forall t \in \left[0, t_f\right] \quad ; \quad u_1(t) \in [0, 1] \tag{34}$$

$$T_3\left(t_f\right) = T_3(0) \tag{35}$$

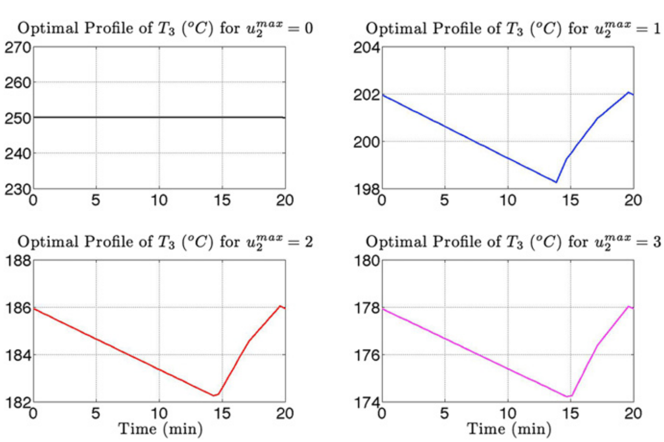

Fig. 7. Details of the optimal oven temperature profiles for different values of the maximum achievable heat transfer improvement $u_2^{\max}$.

Note that in this formulation, the cost function to be minimized is proportional to the energy being consumed with C_1 as factor. This is because the unitary cost (in terms of energy) of the u_2 technology is given as a fraction μ of the unitary cost of the classical heating technology u_1 [see eq. (20) for a reminder of the definition of C_1].

Note also that the last constraint (35) enables a cyclic process to be defined in which the next baking process can begin after the end of the preceding one and starting from the same initial oven temperature $T_3(0)$.

Problem 1 is clearly not a Linear Programming problem neither it is a quadratic problem and this because of the bilinear term $u_2(t) \cdot T_3(t)$ appearing in eq. (32) that involves a product of the control input and a state component. That is the reason why the following equivalent problem is defined in which the following change of the control input is used:

$$\eta_2(t) = \frac{1}{1+u_2(t)} \in \left[\eta_2^{\min}, 1\right] \quad ; \quad \eta_2^{\min} = \frac{1}{1+u_2^{\max}} \tag{36}$$

With this notation, the following equivalent optimization problem can be defined:

Definition 2. (*Problem 2*) *Consider the dynamic model given by*

$$\dot{T}_3 = -\gamma_l \cdot T_3 + \gamma_h \cdot u_1 \tag{37}$$

And let be given:
Two reference profiles $T_2^r(\cdot)$ and $T_3^r(\cdot)$ defined over $[0, t_f]$
A maximum efficiency coefficient $u_2^{\max}$
A coefficient $C_2 = \mu C_1$ that defines the unitary energy cost of the use of u_2
Compute the profiles $u_1^*(\cdot)$ and $\eta_2^*(\cdot)$ as well as the initial values $T_3(0)$ of the oven temperature that minimizes the energy related cost function defined by:

$$J^* = \min_{u_1(\cdot),\, \eta_2(\cdot),\, T_3(0)} \int_0^{t_f} \left[u_1(t) + \mu \cdot \frac{1-\eta_2(t)}{\eta_2(t)}\right]\mathrm{d}t \tag{38}$$

while meeting the following constraints:

$$\forall t \in \left[0, t_f\right] \quad ; \quad T_3(t) - \left(T_3^r(t) - T_2^r(t)\right) \cdot \eta_2 = T_2^r(t) \tag{39}$$

$$\forall t \in \left[0, t_f\right] \quad ; \quad \eta_2(t) \in \left[\frac{1}{1+u_2^{\max}},\ 1\right] \tag{40}$$

$$\forall t \in \left[0, t_f\right] \quad ; \quad u_1(t) \in [0, 1] \tag{41}$$

$$T_3\left(t_f\right) = T_3(0) \tag{42}$$

Table 3
Percentage of energy saving as a function of the jet impingement related heat transfer improvement. Computation is based on a reference scenario in which a constant oven temperature $T_3^r = 250\,^{\circ}\mathrm{C}$ is used in the BONGARD oven corresponding to the thermal characteristics investigated in Section 4.

Jet impingement related coefficient $u_2^{\max}$	Energy saving (%)
0	0
1	9.5
2	12.6
3	14.2
5	15.7
8	16.7
20	17.9

In the new version, the nonlinearity moved from the constraints which are all linear by now to the cost function which now contains the fractional term in η_2. The advantage of this new formulation is that one can use it to compute an upper bound of the potential energy saving that is achievable. Indeed, using $\mu = 0$ in the formulation of Problem 2 yields a LP (Linear Programming) problem that corresponds to the case where the jet impingement has no cost. Thus, the corresponding energy saving obtained by solving the resulting problem would be an upper bound of the energy saving that one would expect when using this technology. LP problems can be solved using now extremely mature tools the details of which are omitted here; interested readers are referred to Ref. [19]. In the next section, the optimal solutions of problem 2 for $\mu = 0$ and different values of the maximal achievable transfer improvement coefficient $u_2^{\max}$ are investigated and commented.

5.4. Estimating an upper bound of energy saving using jet impingement technology

When using $\mu = 0$ in eq. (38), the resulting problem is a Linear Programming problem in the unknown contained in the vector of control input:

$$\tilde{v} = \begin{pmatrix} \tilde{u}_1 \\ \tilde{\eta}_2 \end{pmatrix} = ((u_1(0)\ldots u_1((N-1)\tau))(\eta_1(0)\ldots\eta_2((N-1)\tau)))^T \in I;R^{2N} \tag{43}$$

where $N = t_f/\tau$ is the number of sampling periods. The cost function can be written as a linear function of $\tilde{v}$, namely:

$$J(\tilde{v}) = (\tau \cdot I;\ \ I_N\ \ O;\ \ O_N) \cdot \tilde{v} \tag{44}$$

The inequality constraints (39)–(41) and the equality constraint (42) can be put in the following form:

$$A_{\text{ineq}} \cdot \tilde{v} \prec B_{\text{ineq}} \tag{45}$$

$$B_{\text{eq}} \cdot \tilde{v} = B_{\text{eq}} \tag{46}$$

Using a sampling period of $\tau = 24$ s and a baking scenario of length $t_f = 20 \times 60$ s, an LP problem of dimension 100 is formulated and solved using the MATLAB® LINPROG subroutine. The results are shown in Fig. 6. This figure shows the optimal time profile of the control variables for three different assumptions on the maximum achievable value $u_2^{\max}$ of the heat transfer increase due to jet impingement technology. Fig. 6 also shows the evolution of the corresponding oven temperature T_3. These same profiles are more clearly shown in Fig. 7 where can be clearly seen how the heat transfer improvement enables lower oven temperatures to be used making possible a noticeable energy saving.

In order to compute the corresponding energy saving, the optimal cost given by $J(\tilde{v}^{\text{opt}})$ has to be compared to the reference cost corresponding to the oven being maintained at the reference temperature $T_3^r = 250\ ^\circ\text{C}$ that has been used to define the tracking problem. The reference energy level is then given by

$$J^r = t_f \times \frac{\gamma_l}{\gamma_h} \times T_3^r \tag{47}$$

which leads to the following percentage energy saving computation formula:

$$G = \frac{J^r - J(\tilde{v}^{\text{opt}})}{J^r} \times 100 \tag{48}$$

the results are shown on Table 3 where it can be seen that under the assumption used above, the gain can be close to 16% for a realistic and experimentally validated increase in heat transfer of $u_2^{\max} = 2$.

6. Conclusion and future investigation

This paper is a preliminary investigation of potential energy saving that may be obtained by using jet impingement technology in order to increase the efficiency of the heat transfer between the oven atmosphere and the bread. This is done by first deriving a simple mechanistic model for the French baking process, then by formulating a Linear Programming optimization problem that expresses the tracking of the crust temperature. The basic assumption is that reproducing the temperature profile of the crust guarantees the obtaining of the main properties of the resulting bread regardless our understanding of the complex phenomena that takes place during the baking process.

It remains however true that only the optimistic optimization problem is considered in which the energy cost of the jet impingement technique is negligible when compared to the basic heating process. Under this limiting assumption, an upper bound of expected energy saving has been computed showing an approximated saving level of 16%. Future investigation has to address the non-convex optimization problem in which a non-zero energy cost is used for the jet impingement technology and where an updated heat characteristics corresponding to the future generation of oven are injected. Moreover, the inclusion of radiation terms as well as infrared based technologies in the model and the resulting computation is also an undergoing work.

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