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Brief paper

New path-generation based receding-horizon formulation for constrained stabilization of nonlinear systems☆

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Abstract

In this paper, a new formulation of constrained stabilizing receding-horizon control is proposed. This formulation is based on the use of open-loop steering path generators. The open-loop optimization problem associated to the proposed receding-horizon formulation is scalar in which the optimization variable is the prediction horizon length. Stability is proved in a sampling control scheme. A simple example is given to illustrate the main concepts.

Keywords: Receding horizon; Nonlinear systems; Sampling; Semi-global stabilization; Constrained control; Practical stability

1. Introduction

Receding horizon control has been first recognized as a powerful approach to stabilize nonlinear systems by Keerthi and Gilbert (1988), for discrete-time nonlinear systems and at roughly the same time by Mayne and Michalska (1990) and Michalska and Mayne (1991) for continuous-time nonlinear systems. Since, lots of works have been done to investigate further results in discrete-time framework (Magni, De Nicolao, & Scattolini, 1998; Alamir & Bornard, 1994, 1995b), robustness (De Nicolao, Magni, & Scattolini, 1996; Blauwkamp & Basar, 1999; Yugeng & Xiaojun, 1999; Michalska & Mayne, 1993; Alamir & Bornard, 1995a), discontinuous state feedback formulations (Meadows, Henson, Eaton, & Rawlings, 1995; Michalska, 1995; Marchand, Alamir, & Balloul, 2000), free final time formulations (Michalska, 1997), real time implementability (Ohtsuka & Fujii, 1997; Ohtsuka & Ohata, 1997; Binder et al., 2001), inverse optimality (Magni & Sepulchre, 1998) and hybrid systems related formulations (Parisini & Sacone, 1999). It goes without saying that an exhaustive study of existing works is beyond the scope of this brief paper. Interested readers may consult the excellent recent survey papers (Mayne, Rawlings, Rao, & Scokaert, 2000; Allgower, Badgwell, Qin, Rawlings, & Wright, 1999; Binder et al., 2001).

Beyond the differences in the open-loop optimization problem's formulations, classical formulations share a common feature, namely, the computational difficulty associated to the solution of the open-loop optimization problem. While earlier works used final state equality constraint to insure stability, it is now widely admitted that when optimality is an issue, exact final equality constraints on the state are to be supplanted by a combination of less restrictive inequalities and weighting terms (see for example Michalska & Mayne, 1993; De Nicolao, Magni, & Scattolini, 1998; Chen & Allgower, 1998; Magni, De Nicolao, Magnani, & Scattolini, 2001; Fontes, 2001; Parisini & Zoppoli, 1995; De Nicolao et al., 1996) giving rise to more robust stabilization results associated to easier computations. The aim of the present paper is to show that when only stabilization is addressed, formulations with final state constraint may be of particular interest.

The starting point of this paper lies in the fact that for a wide class of nonlinear systems, open-loop trajectories that steer the system to the origin can be easily generated by exploiting the particular structure of the system (Alamir & Marchand, 1999; Marchand & Alamir, 2003; van Nieuwstadt & Murray, 1998). As an example, in the case

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of the spacecraft in failure mode, the problem amounts to solve 2 nonlinear equations in 2 unknowns (Alamir & Boyer, 2003; Marchand & Alamir, 2003). This fact may question the pertinence of the now widely spread idea according to which formulations with final state constraint are to be unconditionally avoided.

It is worth noting that the approach proposed in this paper is particularly dedicated to the nonlinear stabilization problem. This problem is still a hard task in many cases, especially in the case of nonlinear under-actuated mechanical systems. The counterpart of the fast computation associated to the new formulation proposed in this paper is the loss of optimality and the absence of state-related constraints handling. These two features are ones of the more attractive features in nonlinear predictive control schemes.

The question is then the following: How to use a powerful open-loop path-generation procedure to construct a stabilizing state feedback?

This question is related to a currently active research activity in the nonlinear control community concerning the links between controllability and stabilizability (see for example Clarke, Ledyaev, Sontag, & Subbotin, 1997; Marchand & Alamir, 2000). This paper proposes a set of sufficient conditions enabling an open-loop path-generation procedure to be used to construct a stabilizing state feedback. This leads to a receding-horizon scheme with the following properties: (1) The search space over which the open-loop optimization problem is defined is one-dimensional. The optimization variable being the horizon length. (2) The state feedback law considered in this paper is defined in a sampling scheme. This dramatically simplifies the proof since solutions are defined in the classical sense without need to invoke differential inclusions or solutions in Filippov sense. (3) Since no Lipschitz condition is a priori required, only semi-global stabilization result is obtained.

This paper is organized as follows. First, some definitions, notations and preliminary results are given in Section 2. Section 3 presents the main results of this paper while an illustrative example is proposed in Section 4.

2. Definitions and preliminary results

Let us consider nonlinear systems of the form

$$\dot{x} = f(x,u), \quad x \in \mathbb{R}^n, \; u \in \mathbb{U} \subset \mathbb{R}^m, \tag{1}$$

where $\mathbb{U}$ is a convex and compact subset of $\mathbb{R}^m$ and f is continuous. In what follows, $S(t;x_0;u(.))$ denotes the solution of (1) at instant t starting from the initial condition $x(0) = x_0$ and under the control profile u. For all $r > 0$, $\bar{B}(0,r)$ denotes the closed ball of radius r centered at the origin while $\mathbb{U}^{[0,t_f]}$ is used to designate the set of functions defined on $[0,t_f]$ with values in $\mathbb{U}$. Finally, given any subset $\mathscr{S}$ of an Euclidean space, $\rho(\mathscr{S})$ denotes the radius of $\mathscr{S}$, namely $\sup_{x\in\mathscr{S}} \|x\|$.

The dynamical system (1) is supposed to meet the following assumption:

Assumption 1. For all $\varDelta > 0$, and all piece-wise continuous control profile $u(.)$ entirely contained in $\mathbb{U}$:

$$\lim_{\|x\|\to\infty} \|S(\varDelta;x;u(.))\| = \infty.$$

Assumption 1 simply stipulates that nonzero states cannot be steered infinitely fast to the origin with saturated control. This assumption may exclude some particular systems with backward finite escape time.

Definition 1 (Admissible parameterization of open-loop control profiles). A mapping $\mathscr{U} : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^m$ is said to be an admissible control parameterization map for system (1) if the following conditions hold:

(1) $\mathscr{U}$ is continuous in its arguments.

(2) There is a map $\bar{\varDelta} : \mathbb{R}^n \to \mathbb{R}^+$ and some $\varepsilon > 0$, such that $\mathscr{U}(.;x;\bar{\varDelta}(x)) \in \mathbb{U}^{[0,\bar{\varDelta}(x)]}$ and

$$\|S(\bar{\varDelta}(x);x;\mathscr{U}(.;x;\bar{\varDelta}(x)))\| \leqslant \varepsilon \tag{2}$$

furthermore, $\bar{\varDelta}(.)$ is bounded over bounded sets and such that $\lim_{x\to 0} \bar{\varDelta}(x) = 0$.

(3) For all $\varDelta > 0$, $x \in \mathbb{R}^n$ and all $\tau_1 \leqslant \varDelta$; if $x_{\text{ol}}^{\tau_1}$ is defined by

$$x_{\text{ol}}^{\tau_1}(x,\varDelta) := S(\tau_1;x;\mathscr{U}(.;x;\varDelta)) \tag{3}$$

then, the following holds for all $\tau \in [0, \varDelta - \tau_1]$:

$$\mathscr{U}(\tau;x_{\text{ol}}^{\tau_1}(x,\varDelta);\varDelta-\tau_1) = \mathscr{U}(\tau_1+\tau;x;\varDelta). \tag{4}$$

Remark 1. Assumption (2) is rather restrictive when $\bar{\varDelta}(.)$ is supposed to be defined over all $\mathbb{R}^n$. Indeed it assumes that however far the initial state may be, it can be steered close to the origin using bounded controls. This is clearly unrealistic for general nonlinear systems. This is the reason why this assumption is relaxed in Corollary 1 to yield a more natural assumption. At the time being, this assumption is maintained for simplicity of exposition.

As for Assumption (4), it is a kind of transitivity condition recalling the Bellman's principle. It only states that when interrupting an open-loop trajectory at some state $x_{\text{ol}}^{\tau_1}$, the parameterized open-loop control starting from $x_{\text{ol}}^{\tau_1}$ and with as parameter the remaining delay $\varDelta - \tau_1$ is nothing but the remainder of the original one.

Lemma 2. *Assume that an admissible control parameterization map $\mathscr{U}$ exists for system (1) then the following function is well defined for all $\alpha > 0$ and all compact sets $\mathscr{X}$ containing the closed ball $\bar{B}(0,\varepsilon)$:*

$$\hat{\varDelta}(x,\mathscr{X}) := \operatorname{Arg}\min_{\varDelta\geqslant 0} \varphi(\varDelta,x)$$

$$\textit{under } g(\varDelta,x) \in \mathscr{X} \textit{ and } \mathscr{U}(.;x;\varDelta) \in \mathbb{U}^{[0,\varDelta]}, \tag{5}$$

where

$$\varphi(\varDelta,x) := \|S(\varDelta;x;\mathscr{U}(.;x;\varDelta))\|^2 + \alpha\varDelta \tag{6}$$

and

$$g(\varDelta,x) := S(\varDelta;x;\mathscr{U}(.;x;\varDelta)). \tag{7}$$

Proof. This immediately comes from the fact that the minimized function is continuous in Δ and that, according to Assumption (2), $\Delta = \bar{\Delta}(x)$ is always an admissible value with as corresponding cost $\varepsilon^2 + \alpha\bar{\Delta}(x)$. Therefore, the optimization problem (5) admits the same solution than the following one:

$$\hat{\Delta}(x, \mathscr{X}) := \operatorname{Arg} \min_{\Delta \in \mathscr{C}(x,\mathscr{X})} \varphi(\Delta, x),$$

where $\mathscr{C}(x, \mathscr{X})$ is the non-empty compact set defined by $\mathscr{C}(x, \mathscr{X}) := \{\Delta \geqslant 0 \text{ s.t. } \varphi(\Delta, x) \leqslant \varepsilon^2 + \alpha\bar{\Delta}(x),\ g(\Delta, x) \in \mathscr{X}$ and $\mathscr{U}(.; u; \Delta) \in \mathbb{U}^{[0,\Delta]}\}$. □

Remark 3. It is very important to note that $\mathscr{X}$ needs not to be a small neighborhood around the origin, it is even unnecessary that the radius of $\mathscr{X}$ be strictly lower than the norm of the initial state x.

Definition 2 (Definition of the feedback law). Assume that $\mathscr{U}$ is an admissible control parameterization map for system (1). Let be given a compact set $\mathscr{X}$ containing the closed ball $\bar{B}(0, \varepsilon)$. Let $\sigma > 0$ be some sampling time. Consider the following dynamic state feedback (where for any integer k, $v(k)$ is used to designate $v(k\sigma)$ for any time-varying quantity v)

$$\Delta(k) = \max\{\sigma, \hat{\Delta}(x(k), \mathscr{X})\}, \tag{8}$$

$$u(k\sigma + \tau) = \mathscr{U}(\tau; x(k); \Delta(k)); \quad 0 \leqslant \tau \leqslant \sigma \tag{9}$$

namely, Δ is a controller's internal state whose evolution in time is piece-wise constant with (8) as updating rule.

3. Main result

In this section, a state feedback is said to stabilize an ε-neighborhood of the origin if it stabilizes some neighborhood of the origin contained in $\bar{B}(0, r(\varepsilon))$ with $r(\varepsilon) = O(\varepsilon)$.

Theorem 1 (Main result). *Assume that an admissible parameterization map $\mathscr{U}$ exists for system* (1). *For all compact sets $\mathscr{X} \subset \mathbb{R}^n$ containing the closed ball $\bar{B}(0, 2\varepsilon)$, there is sufficiently small $\alpha > 0$ and $\sigma > 0$ such that the feedback given in Definition* 2 *asymptotically stabilizes* (*in the Lyapunov sense*) *an ε-neighborhood of the origin with controls that remains in $\mathbb{U}$ and this for all initial states that belong to $\mathscr{X}$.*

Proof. It will be proved that $V(x) := \varphi(\hat{\Delta}(x, \mathscr{X}), x)$, where $\varphi(.)$ is given by (6)–(7) is a suitable Lyapunov function for the stabilization problem under consideration for sufficiently small $\sigma > 0$ and for all $\alpha > 0$ satisfying

$$\alpha \leqslant \frac{\varepsilon^2}{\sup_{x \in \bar{B}(0,\rho(\mathscr{X})+1)} \bar{\Delta}(x)}. \tag{10}$$

- Note first that V is continuous being the minimum of a continuous function over compact sets. It is also proper in x in the sense that $\lim_{\|x\| \to \infty} V(x) = \infty$ thanks to Assumption 1. Furthermore, $V(0) = 0$.
- For all $x \in \mathscr{X}$, one has according to Assumption (2):

$$V(x) \leqslant \varepsilon^2 + \alpha \sup_{x \in \mathscr{X}} \bar{\Delta}(x) =: \bar{V}(\mathscr{X}). \tag{11}$$

- Let $\bar{\mathscr{X}}$ denote the following compact set:

$$\bar{\mathscr{X}} := \{x \in \mathbb{R}^n \mid V(x) \leqslant \bar{V}(\mathscr{X})\}. \tag{12}$$

- If it can be proved that for sufficiently small $\sigma > 0$, $V(x(.))$ is nonincreasing over closed-loop trajectories that remain in $\bar{\mathscr{X}}$ then, by induction based on the definition of $\bar{\mathscr{X}}$, it can be stated that for such σ's, all closed-loop trajectories starting in $\mathscr{X}$ are entirely contained in $\bar{\mathscr{X}}$ and hence, corresponding Lipschitz-like constants can be used. (Such constants are necessary to introduce $M_1(\mathscr{X})$ and $M_2(\mathscr{X})$ hereafter.)
- Let us then consider $x_{\mathrm{cl}}(.)$ to be a closed-loop trajectory entirely contained in $\bar{\mathscr{X}}$ and let us prove that $V(x_{\mathrm{cl}}(.))$ is nonincreasing for sufficiently small sampling time $\sigma > 0$.
- Consider a time interval $[k\sigma, (k+1)\sigma]$ such that $\Delta(k\sigma) \geqslant \sigma$. Consider, in accordance with the notation of Definition 1, the open-loop trajectory $x_{\mathrm{ol}}^{\tau}(x(k\sigma), \Delta(k\sigma))$ for $\tau \in [0, \sigma]$. It is then an immediate consequence of (4) and the very definition of the feedback law that for all $\tau \in [0, \sigma[$ and as long as $\Delta(k\sigma) \geqslant \sigma$

$$V(x_{\mathrm{ol}}^{\tau}(x(k\sigma), \Delta(k\sigma))) \leqslant V(x(k\sigma)) - \alpha\tau \tag{13}$$

since for all $\tau \in [0, \sigma]$, $\Delta(k\sigma) - \tau$ is a candidate admissible sub-optimal solution of $\min_{g(\Delta, x_{\mathrm{ol}}^{\tau}) \in \mathscr{X}} \varphi(\Delta, x_{\mathrm{ol}}^{\tau})$ with $V(x(k\sigma)) - \alpha\tau$ as corresponding sub-optimal cost. Now, by definition of the inter-sampling feedback law, one clearly has

$$x_{\mathrm{cl}}(k\sigma + \tau) = x_{\mathrm{ol}}^{\tau}(x(k\sigma), \Delta(k\sigma)), \quad \forall \tau \in [0, \sigma[.$$

This with (13)

$$V(x_{\mathrm{cl}}(k\sigma + \tau)) \leqslant V(x(k\sigma)) - \alpha\tau$$
$$\text{whenever } \Delta(k\sigma) \geqslant \sigma. \tag{14}$$

- Therefore, a closed-loop trajectory that starts in $\mathscr{X}$ remains in $\bar{\mathscr{X}}$ at least as long as $\Delta(k\sigma) \geqslant \sigma$ and moreover, it satisfies inequality (14).
- But inequality (14) clearly shows that $\Delta(k\sigma)$ cannot remain indefinitely $\geqslant \sigma$. Therefore, there is $k_0 \in \mathbb{N}$ such that $\Delta(k_0\sigma) = \Delta_0$ with Δ_0 satisfying the following conditions:

$$\Delta_0 < \sigma, \tag{15}$$

$$\|S(\Delta_0; x(k_0\sigma); \mathscr{U}(.; x(k_0\sigma); \Delta_0))\|^2 + \alpha\Delta_0 \leqslant \varepsilon^2 + \alpha\bar{\Delta}(x(k_0\sigma)), \tag{16}$$

$$S(\Delta_0; x(k_0\sigma); \mathscr{U}(.; x(k_0\sigma); \Delta_0)) \in \mathscr{X}. \tag{17}$$

Indeed, (16) holds because $\bar{\Delta}(x(k_0\sigma))$ is a sub-optimal solution while (17) holds by definition of the constrained optimization problem (5) defined at $x(k_0\sigma)$.

- Therefore, $S(.;x(k_0\sigma);\mathscr{U}(.;x(k_0\sigma);\Delta_0))$ is a system trajectory under bounded control over a time interval of length lower than σ with end-point lying in $\mathscr{X}$. Hence, there is $M_2(\mathscr{X})$ such that (see (16))

$$\|x(k_0\sigma)\|^2 \leqslant \varepsilon^2 + \alpha\bar{\Delta}(x(k_0\sigma)) + M_2(\mathscr{X})\sigma. \quad (18)$$

  Therefore, for sufficiently small σ, one has $x(k_0\sigma)\in\bar{B}(0,\rho(\mathscr{X})+1)$ which, with (18) and (10) yields $\|x(k_0)\|^2 \leqslant 4\varepsilon^2$. In particular, $x(k_0\sigma)\in\bar{B}(0,2\varepsilon)\subset\mathscr{X}\subset\bar{\mathscr{X}}$.
- To sum up, for sufficiently small $\sigma>0$, every closed-loop trajectory that starts in $\mathscr{X}$ remains in $\bar{\mathscr{X}}$ and the use of Lipschitz like constants over $\bar{\mathscr{X}}$ is a posteriori justified. Furthermore, $x(k)$ enters $\bar{B}(0,2\varepsilon)$ after a finite time and therefore, the semi-globally stabilized ε-neighborhood of the origin is readily given by

$$\left\{x\in\mathbb{R}^n \,|\, V(x)\leqslant\varepsilon^2+\sup_{\|x\|\leqslant 2\varepsilon}\bar{\Delta}(x)\right\}. \qquad \square$$

As it is mentioned in Remark 1, Condition (2) in the definition of admissible control parameterization needs to be relaxed. The following is a straightforward corollary of Theorem 1.

Corollary 1. *If, in Definition* 1 *of admissible control parameterization, Condition* (2) *is replaced by the following condition*:

There is a map $\bar{\Delta}:\mathscr{C}\subset\mathbb{R}^n\to\mathbb{R}^+$ [*where* $\mathscr{C}$ *is an open bounded set containing the origin*] *such that*

$$\|S(\bar{\Delta}(x);x;\mathscr{U}(.;x;\bar{\Delta}(x)))\|=0$$

and

$$\mathscr{U}(.;x;\bar{\Delta}(x))\in\mathbb{U}^{[0,\bar{\Delta}(x)]}$$

furthermore, $\bar{\Delta}(.)$ *is bounded over bounded sets and such that* $\lim_{x\to 0}\bar{\Delta}(x)=0$.

Then there is a compact set $\mathscr{X}$ *w.r.t which the semi-global stabilization result of Theorem* 1 *holds with* $\varepsilon=0$. *Furthermore*: $\rho(\mathscr{X})\to\infty$ *when* $\rho(\mathscr{C})\to\infty$.

Proof. It is sufficient to prove that there is a compact set $\mathscr{X}$ such that the corresponding $\bar{\mathscr{X}}$ [see (12)] is contained in $\mathscr{C}$ that is

$$\{x\in\mathbb{R}^n \mid V(x)\leqslant\bar{V}(\mathscr{X})\}\subset\mathscr{C}, \quad (19)$$

where

$$\bar{V}(\mathscr{X}):=\alpha\sup_{x\in\mathscr{X}}\bar{\Delta}(x) \quad (20)$$

but ($\mathscr{X}:=\bar{B}(0,\eta),\varepsilon<\eta/2$) is clearly a suitable choice whenever $\eta>0$ is such that

$$\psi(\eta):=\alpha\sup_{\|x\|\leqslant\eta}\bar{\Delta}(x)<\min_{x\in\partial\mathscr{C}}V(x)\neq 0 \quad (21)$$

such $\eta>0$ clearly exists since $\bar{\Delta}(x)\to 0$ when $x\to 0$. [Note that $\min_{x\in\partial\mathscr{C}}V(x)\neq 0$ is implied by the definition of $V(x)$ because $\min_{x\in\partial\mathscr{C}}\|x\|=:\rho(\mathscr{C})\neq 0$ and states such that $\|x\|\geqslant\rho(\mathscr{C})$ cannot be steered to 0 infinitely fast with bounded control].

Now when $\rho(\mathscr{C})$ goes to infinity, the r.h.s of (21) does the same (V is proper) and since $\bar{\Delta}(.)$ is bounded over bounded set, this means that one can take $\eta:=\rho(\mathscr{X})$ going to infinity. $\square$

4. Illustrative example: Kawski's system

Let us consider the following system proposed by Kawski (1990):

$$\dot{x}_1=u, \quad \dot{x}_2=x_2-x_1^3, \quad u\in[-\bar{u},\bar{u}]. \quad (22)$$

It will be first shown that system (22) meets the requirements of Assumption 1. For, let $\Delta>0$ be some fixed horizon length and let $u(.)$ be any bounded admissible control profile. It is then clear that

$$\lim_{|x_1(0)|\to\infty}\|S(\Delta;x(0);u(.))\|\geqslant|x_1(0)|-|\bar{u}\delta|\to\infty.$$

Therefore, to prove that Assumption 1 holds for the Kawski's system, one has to prove that given any bound $\rho>0$ on the initial state $x_1(0)$, the following holds:

$$\lim_{|x_2(0)|\to\infty,|x_1(0)|\leqslant\rho}\|S(\Delta;x(0);u(.))\|=\infty \quad (23)$$

which is clearly true since as soon as $|x_2(0)|$ becomes greater that ρ^3, the future system's trajectory is such that $|x_2(t)|>|x_2(0)|$ for all $t>0$, therefore $\|S(\Delta;x(0);u(.))\|\geqslant|x_2(0)|\to\infty$ when $x_2(0)\to\infty$. Therefore, Assumption 1 holds for the Kawski's system (22).

The definition of $\mathscr{U}(\tau;x_0;\Delta)$ is obtained using the following steps:

(1) Let us first decide to search for an open-loop control of the form

$$u(\tau):=a+b\tau+c\tau^2, \quad \tau\in[0,\Delta] \quad (24)$$

such that the open-loop trajectory satisfies

$$u(\Delta)=S_1(\Delta;x;u(.))=S_2(\Delta;x;u(.))=0.$$

(2) The first two constraints $u(\Delta)=S_1(\Delta;x;u(.))=0$ lead to:

$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}1&\Delta\\1&\frac{\Delta}{2}\end{pmatrix}^{-1}\left[\begin{pmatrix}0\\-x_1/\Delta\end{pmatrix}-\begin{pmatrix}1\\\frac{1}{3}\end{pmatrix}\Delta^2c\right] \quad (25)$$

which is well defined for all $\Delta\geqslant\sigma>0$.

(3) With (25) in hand, the control is function of the only c. Consider then the scalar equation (defined for fixed x and $\Delta\geqslant\sigma$) giving the final state x_2

$$F_2(c):=S_2(\Delta;x;u(.))=0$$

and denote its solution by $\hat{c}(x,\Delta)$.

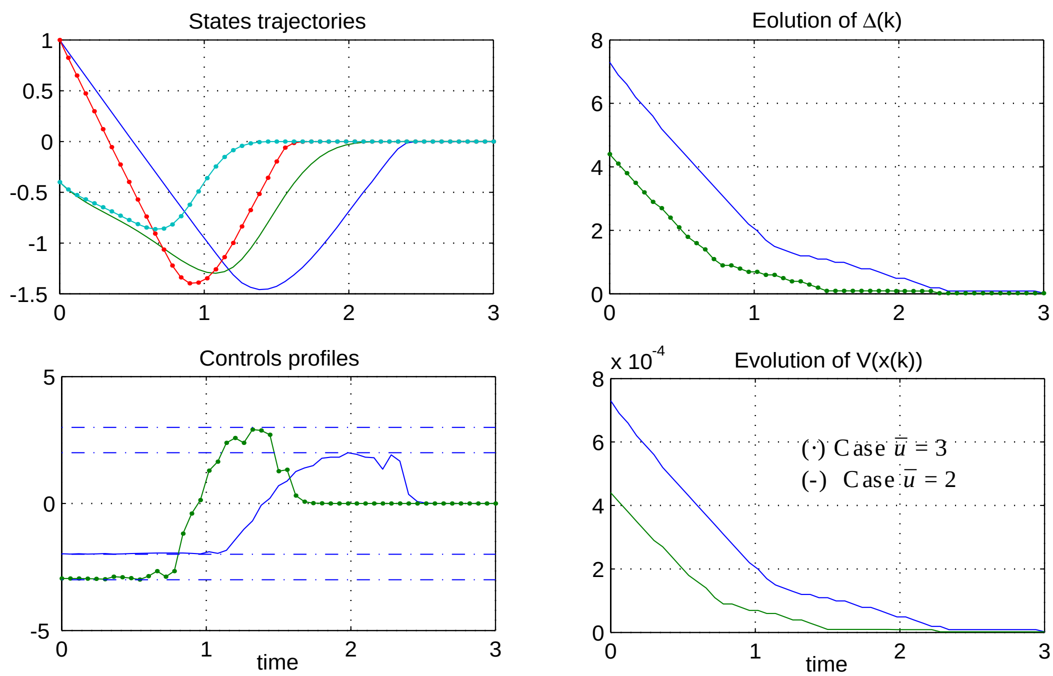

Fig. 1. Simulation results for: $\sigma = 0.02$, $\alpha = 0.001$, $\mathscr{X} := [-2, 2]^2$ and different admissible control levels $\bar{u} \in \{2, 3\}$.

(4) The parameterization is then completely defined for all $\Delta \geqslant \sigma$ by

$$\mathscr{U}(\tau; x; \Delta) := a + b\tau + \hat{c}(x, \Delta)\tau^2, \tag{26}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} := \begin{pmatrix} 1 & \Delta \\ 1 & \frac{\Delta}{2} \end{pmatrix}^{-1} \times \left[\begin{pmatrix} 0 \\ -x_1/\Delta \end{pmatrix} - \begin{pmatrix} 1 \\ \frac{1}{3} \end{pmatrix} \Delta^2 \hat{c}(x, \Delta) \right]. \tag{27}$$

(5) As for $\Delta < \sigma$, take $\mathscr{U}(\tau; x; \Delta) := \mathscr{U}(\tau; x; \sigma)$.

It is then clear that for a given saturation level $\bar{u}$, there is a bounded set of initial states [$\mathscr{C}$ in Corollary 1] such that the resulting open-loop control defined by $\mathscr{U}(\tau; x; \Delta)$ satisfies the saturation constraint for sufficiently high $\Delta > 0$. Therefore, Condition (2′) of the definition of admissible control parameterization invoked in Corollary 1 holds. Moreover, since polynomial open-loop profiles are used, Condition (4) is a direct consequence of the fact that any portion of a polynomial trajectories is a polynomial trajectory of the same degree.

It is important to point out the fact that the possibility of bringing the problem of admissible control parameterization to a quite low order problem (one-dimensional here) is not necessarily linked to the simplicity of the Kawski's system considered here. It can be shown that the same can be done for much more complicated systems such as the satellite in failure mode (6 states, 2 controls), the ball and beam example (4 state, 1 control), the inverted pendulum and many other systems (see Alamir & Marchand, 1999; Alamir & Boyer, 2003; Marchand & Alamir, 2003). Indeed, in these last three examples, the parameterization reduces to the solution of 2 nonlinear equations in 2 unknowns. Showing this however would be longer to explain compelling the use of a simple example here for pure illustrative purposes.

In the following simulations, the following first order interpolation scheme has been used to implement the feedback law (9) for $0 \leqslant \tau \leqslant \sigma$:

$$u(k\sigma + \tau) = \left(1 - \frac{\tau}{\sigma}\right) \mathscr{U}(0; x(k); \Delta(k)) + \frac{\tau}{\sigma} \mathscr{U}(\sigma; x(k); \Delta(k)).$$

Fig. 1 shows the closed-loop behavior in two cases. The first with the control saturation $\bar{u} = 2$ and the second with $\bar{u} = 3$. The set $\mathscr{X}$ used in the definition of the optimal control (5) is taken equal to $\mathscr{X} := [-2, 2]^2$. Note how the state feedback control is automatically adapted to meet the saturation constraint leading to a faster stabilization time when higher admissible control level $\bar{u}$ allows it. As for the choice of the control parameter $\alpha > 0$, experience shows that its value's choice is not critical since many different values have been successfully used. Note finally that the "*nervous*" behavior of the closed-loop control near the control limit is most probably due to numerical stopping criteria in the scalar optimization problem solver.

5. Conclusion

In this paper, a new stabilizing receding-horizon control formulation is proposed. This formulation is based on open-loop path generation parameterization. The stabilizing result is given in a sampling control scheme leading to semi-global stabilization. The formulation proposed in this paper is particularly adapted to systems for which the generation of open-loop steering trajectories is easy to obtain. It gives a theoretical framework to use such trajectory generators to construct stabilizing state feedback.

References

Alamir, M., & Bornard, G. (1994). On the stability of receding horizon control of nonlinear discrete-time systems. *Systems & Control Letters*, *23*, 291–296.

Alamir, M., & Bornard, G. (1995a). Optimization based stabilizing strategy for nonlinear discrete time systems with unmatched uncertainties. In *Proceedings of the second international symposium on methods and models in automation and robotics*, Miedzyzdroje, Poland (pp. 193–198).

Alamir, M., & Bornard, G. (1995b). Stability of a truncated infinite constrained receding horizon scheme: The general nonlinear case. *Automatica*, *31*(9), 1353–1356.

Alamir, M., & Boyer, F. (2003). Fast generation of attractive trajectories for an under-actuated satellite: Application to feedback control design. *Journal of Optimization in Engineering*, *4*, 215–230.

Alamir, M., & Marchand, N. (1999). Numerical stabilization of nonlinear systems—exact theory and approximate numerical implementation. *European Journal of Control*, *5*, 87–97.

Allgower, F., Badgwell, T. A., Qin, J. S., Rawlings, J. B., & Wright, S. J. (1999). Nonlinear predictive control and moving horizon estimation. an introductory overview. In P. M. Frank (Ed.), *Advances in control highlights of ECC'99*, London, UK: Springer–Verlag (pp. 81–93).

Binder, T., Blank, L., Bock, H. G., Burlisch, R., Dahmen, W., Kronseder, T., Marquardt, W., Diehl, M., Schloder, J. P., & von Stryk, O. O. (2001). In M. Grotschel, & Krumke, & J. Rambau (Eds.), *Introduction to model based optimization of chemical processes on moving horizons, online optimization of large scale systems: State of the art*. Berlin: Springer.

Blauwkamp, R., & Basar, T. (1999). A receding-horizon approach to robust output feedback control for nonlinear systems. In *Proceedings of the 38th IEEE conference on decision and control*, Vol. 5, NJ, USA (pp. 4879–4884).

Chen, H., & Allgower, F. (1998). Quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. *Automatica*, *34*(10), 1205–1218.

Clarke, F. H., Ledyaev, Y. S., Sontag, E. D., & Subbotin, A. I. (1997). Asymptotic controllability implies feedback stabilization. *IEEE Transactions on Automatic Control*, *42*(10), 1394–1407.

De Nicolao, G., Magni, L., & Scattolini, R. (1996). On the robustness of receding-horizon control with terminal constraints. *IEEE Transactions on Automatic Control*, *41*(3), 451–453.

De Nicolao, G., Magni, L., & Scattolini, R. (1998). Stabilizing receding-horizon control of nonlinear time-varying systems. *IEEE Transactions on Automatic Control*, *43*(7), 1030–1036.

Fontes, F. A. C. C. (2001). A general framework to design stabilizing nonlinear model predictive controllers. *Systems & Control Letters*, *42*(2), 127–143.

Kawski, M. (1990). Stabilisation of nonlinear systems in the plane. *Systems & Control Letters*, *12*, 169–175.

Keerthi, S. S., & Gilbert, E. G. (1988). Optimal infinite horizon feedback laws for a general class of constrained discrete time systems. *Journal of Optimization Theory Applications*, *57*, 265–293.

Magni, L., De Nicolao, G., Magnani, L., & Scattolini, R. (2001). A stabilizing model-based predictive control algorithm for nonlinear systems. *Automatica*, *37*(9), 1351–1362.

Magni, L., De Nicolao G., & Scattolini, R. (1998). Output feedback receding horizon control of discrete-time nonlinear systems. In *Proceedings of the 4th IFAC nonlinear control systems design symposium*, Oxford, UK (pp. 422–427).

Magni, L., & Sepulchre, R. (1998). Stability margins of nonlinear receding horizon control via inverse optimality. *Systems & Control Letters*, *32*(4), 241–245.

Marchand, N., & Alamir, M. (2000). Asymptotic controllability implies feedback stabilizability: A more general result. In *Lecture notes in control and information sciences*, *NCN Springer Series* (pp. 81–95). Berlin: Springer.

Marchand, N., & Alamir, M. (2003). Numerical stabilisation of a rigid spacecraft with two actuators. *Journal of Dynamic Systems, Measurement and Control*, *125*, 489–491.

Marchand, N., Alamir, M., & Balloul, I. (2000). Stabilization of nonlinear systems by discontinuous state feedback. In *Lecture notes in control and information sciences*, *NCN Springer Series* (pp. 81–93). Berlin: Springer.

Mayne, D. Q., & Michalska, H. (1990). Receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, *35*, 814–824.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., & Scokaert, P. O. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, *36*, 789–814.

Meadows, E. S., Henson, M. A., Eaton, J. W., & Rawlings, J. B. (1995). Receding horizon control and discontinuous state feedback stabilization. *International Journal of Control*, *62*(5), 1217–1229.

Michalska, H. (1995). Discontinuous receding horizon control with state constraints. In *Proceedings of the 1995 American control conference*, Vol. 5, IL, USA (pp. 3500–3504).

Michalska, H. (1997). A new formulation of receding horizon stabilizing control without terminal constraint on the state. *European Journal of Control*, *3*(1), 2–14.

Michalska, H., & Mayne, D. Q. (1991). Receding horizon control of nonlinear systems without differentiability of optimal value function. *Systems & Control Letters*, *16*, 123–130.

Michalska, H., & Mayne, D. Q. (1993). Robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control*, *38*, 1623–1633.

Ohtsuka, T., & Fujii, H. (1997). Real-time optimization algorithm for nonlinear receding horizon control. *Automatica*, *33*(6), 1147–1154.

Ohtsuka, T., & Ohata, K. (1997). Nonlinear receding horizon output feedback control. In *Proceedings of the 36th IEEE conference on decision and control*, Vol. 5, San Diego, USA (pp. 4332–4333).

Parisini, T., & Sacone, S. (1999). A hybrid receding horizon control scheme for nonlinear systems. *Systems & Control Letters*, *38*(3), 187–196.

Parisini, T., & Zoppoli, R. (1995). A receding-horizon regulator for nonlinear systems and a neural approximation. *Automatica*, *31*(10), 1443–1451.

van Nieuwstadt, M. J., & Murray, R. M. (1998). Real-time trajectory generation for differentially flat systems. *International Journal of Robust Nonlinear Control*, *8*(11), 995–1020.

Yugeng, X., & Xiaojun, G. (1999). Properties of receding horizon control for nonlinear systems. *Control Theory & Applications*, *16*, 118–123.

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