Distributed constrained model predictive control for energy management in buildings
Part 1: Zone Model Predictive Control

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Abstract

In this paper, a distributed model predictive control to manage the actuation of the whole actuators (heating/cooling, ventilation, lighting, shading) in a multi-zone building to control comfort parameters (temperature, indoor CO_2 level and indoor illuminance). The control process is performed in a distributed fashion and handle variable prices as well as resource limitation in a context of a multi energy source building. To this end, we firstly present a zonal non linear model predictive controller which is concerned by zonal decision making, we then provide a coordination scheme based on a primal decomposition to adresse the resource allocation problem that happens in the presence of global constraints on power consumption. We finally provide some simulation results attesting the fast convergence of control algorithm and the benefit of the controller.

Keywords: Distributed Model Predictive Control; Constrained control; Building Management System; Smart buildings.

1. Introduction

1.1. Predictive control for building energy management- a brief literature review

Due to the huge consumption of building area that represents no less that 40% of total primal energy consumed in the world and the widely carried out CO_2 emission reduction policy, a great interest of the research community -especially in the last decade- have lead to a better comprehension and qualification of the issues related to building energy management.

In spite the fact that many control strategies have been investigated, the problem of energy management in buildings remains essentially open, as it is attested by [7] where a good overview of the principle advanced control techniques that were studied until now and a discussion on conventional control, namely the so called building energy management systems (**BEMS**) are presented.

An exhaustive review of the existing control strategies and the ones proposed in literature goes beyond the scope of this article, nevertheless it is important to point out the emergence of model predictive control as a particularly adapted approach for building energy management [5], some crucial benefits of this paradigm lies in the following:

- Taking explicitly into account dynamical characteristics as well as: weather forecast, price variations, constraints on resource limitation, multi-sources systems, etc. to optimally schedule actuators operations in building.
- Handling multi-inputs multi-outputs systems, which is the case in buildings.
- Gives coherence in the process of decision making. Contrarily, the conventional "rule-based approach" (or expert system) (e.g: [10])leads generally to a complex logical tree structure which remains intractable when the number of rules and tuning parameters become high.
- Handling economical objectives (variable price for example), this can be very difficult and impractical using the commonly used "rule-based approach". Many studies (e.g: [28, 10, 11]) propose using model predictive control to overcome this issue and demonstrate the economical benefit of using this technique.

A variety of applications of model predictive control in energy related issues for buildings have been proposed. To cite only a few of them, one can mention: [21] where a stochastic predictive control has been designed to control to heating of high inertia buildings. A control of thermal energy storage in building cooling systems has been proposed by [2] while [4] proposed a management of polygeneration systems with predictive technique. In [19] a distributed predictive control strategy is applied to the thermal regulation of buildings. The reader can refer to [14, 3, 9, 13, 12, 8, 28, 1, 6, 20, 17] for more literature on the topic. As mentioned, model predictive control for building energy management has been widely studied. However, one can mention that most studies:

- Focused generally on the thermal aspect,
- Don't consider equipments with nonlinear characteristics,
- Generally consider one zone of a building and hence don't integrate in the decision process the possible resource limitations or shared actuators that go beyond the scope of the zone;
- Propose generally a centralized control scheme, which is unrealistic and unsafe when the number of zones in the building become high.

1.2. Our contribution

To overcome the limitations cited above, we propose in this paper the design of a fully distributed predictive control scheme to control the indoor conditions in each zone, namely we are interested in controlling simultaneously : indoor temperature, indoor CO_2 level and indoor illuminance by controlling all the actuators of the zone (HVAC, lighting, , shading). This problem has been studied (see for e.g: [22]), it has been proposed to use a SLP (Sequential Linear Programming) to tackle the non linear programming resulting from the zone predictive control based an hourly nonlinear model; contrarily to the SLP used [22], we propose in our paper a simpler procedure which derivative free, the model used in our work is a minute sampled nonlinear model which has the benefit of giving more reactivity on short term decision making.

Moreover, we consider the problem of a global resource limitation on the building, which means a total maximum power available of different power sources that can have different dynamic rates (e.g: power from grid + local solar production), in this context the zones decisions can no longer be done independently, to tackle this issue a coordination scheme based on an optimal resource allocation is then proposed. The proposed procedure has the nice feature of proving a sequence of feasible iterates which is a crucial point as mentioned in [26].

The simultaneous handling of these features represents the originality of our contribution.

The paper is organized as follows: In 2 brief recalls on model predictive control and distributed model predictive control are provided, 3 describes clearly the problem and the algorithm used to solve the MPC related optimization problem. Simulation results are finally provided in section 4.

2. Recalls on Nonlinear MPC

Consider the following general nonlinear dynamical system:

$$x_{k+1} = f(x_k, u_k, w_k), y_k = h(x_k, u_k, w_k) \tag{1}$$

Where: $(x, u, w) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w}$ are respectively the state, input and disturbance vectors. k is the time.

Let us first define the following notations:

$$\mathbf{u}_k = [u_k^t, \ldots, u_{k+N-1}^t]^t \tag{2a}$$
$$\mathbf{y}_k = [y_k^t, \ldots, y_{k+N-1}^t]^t \tag{2b}$$
$$\mathbf{w}_k = [w_k^t, \ldots, w_{k+N-1}^t]^t \tag{2c}$$

In model predictive control a representation of the process (1) as well as a description of the future perturbation $\mathbf{w}_k$ is used in order to find at each decision instant k the best open-loop control sequence $\mathbf{u}_k^\star$ according the some objective function $J(\mathbf{u}_k, x_k)$. Only the first component of the optimal control sequence i.e: u_k is applied. Namely, the following optimization problem has to be solved:

$$\mathbf{u}_k^\star = \underset{\mathbf{u}}{\mathbf{Argmin}}\ J(\mathbf{u}, \mathbf{y}) \quad \text{s.t.} \quad (1) \text{ and } C^{st}(\mathbf{y}, \mathbf{w}, \mathbf{u}, x_k) \leq 0 \tag{3}$$

where C^{st} are a set of operational constraints that have to hold. The whole procedure is repeated at the next sampling time $k+1$, based on new state measurement or observation of x_{k+1}. The reader can find more details in [18]

It goes without saying that the complexity of the optimization problem (3) is one of the most important issue we are facing in model predictive control, since this optimization procedure has to be carried out on line at each sampling time.

3. Zone Model Predictive Control

3.1. Problem description

We are interested in providing comfort to occupants at the lower energetic cost, the energetic cost has to be interpreted depending on the context, to be the total amount of energy consumed in the case of the constant price tarification or to be the energetic invoice in the case of variable energy tarification. In this section, we design a model predictive controller for zonal environmental conditions, namely: indoor temperature, CO_2 level and indoor illuminance. For the moment all interactions between zones are neglected.

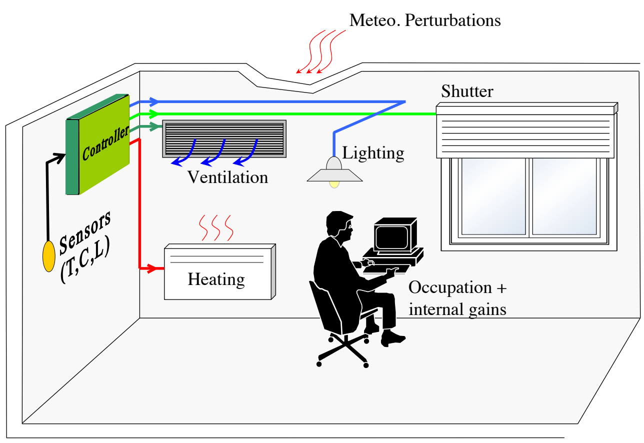

Figure 1: Piece-wise linear approximation of ϕ^N.

Variables	Description	Unit
u_w	FCU valve opening	$[-]$
u_f	FCU fan speed	$[-]$
u_h	Heating ctrl	$[-]$
u_v	Ventilation control	$[-]$
u_l	Lighting control	$[-]$
$\{u_b^i\}_{i=1,\dots,N_f}$	Blind ctrl facade i	$[-]$
T_w	Inlet FCU water temp.	$[^oC]$
T_{ex}	Outdoor temperature	$[^oC]$
T_{adj}	Adjacent zones temp.	$[^oC]$
$\{\phi_g^i\}_{i=1,\dots,N_f}$	Global irr. flux facade i	$[W/m^2]$
Occ	Number of occupants	$[-]$
C_{ex}	Outdoor CO_2 level	$[ppm]$
T	Indoor air temperature	$[^oC]$
C	Indoor CO_2 level	$[ppm]$
L	Indoor illuminance	$[Lux]$

Table 1: Description of Input/Output and exogenous variables

3.2. Zone modelling

3.2.1. Zone dynamical model

For the sake of conciseness, all the details concerning the physical aspects modelling of the zone are omitted, one can refer to [25] for more information regarding building's dynamical modelling. Therefore only the key structural feature of the identified dynamic model of a building zone are presented since they are needed to understand the fixed-point iteration used later in deriving the solution of the MPC related optimization problem. However, the complete details of the model are omitted since they result from classical arguments that can be found in existing references [23, 27, 15].

The identification of a structured model involving the key zone quantities leads to the following state space dynamic representation:

$$\left\{\begin{array}{lll} x^+ & = & A\cdot x + \dots \\ & & B^{\text{fcu}}(T_w - T)\phi^N(u_w,u_f) + B^{u_h}u_h + \dots \\ & & B_T^{u_v}(T_{ex}-T)u_v + B_C^{u_v}(C^{ex}-C)u_v + \dots \\ & & B^{u_l}u_l + \sum_{i=1}^{i=N_f} B^i_{u_b}\phi^i_g u^i_b + \dots \\ & & \sum_{i=1}^{i=N_f} F^{\phi^g_i}\phi^g_i + F^{T_{ex}}T_{ex} + F^{occ}Occ + \dots \\ & & \sum_{i=1}^{i=N_{adj}} F^{T^i_{adj}} T^i_{adj} \\ y & = & C\cdot x + \sum_{i=1}^{i=N_f} D^i_{u_b}\phi^i_g u^i_b + D^{u_l}u_l \end{array}\right. \tag{4}$$

where x is the state of the identified model, $y = (T, C, L)^t \in \mathbb{R}^3$ is the output vector which gathers the physical key quantities, namely, the indoor temperature T, the CO_2 level C and the indoor illuminance level L. $u \in \mathbb{R}^n_u$ regroups all the control input variables, namely $u = (u_w, u_f, u_h, u_c, u_v, u_l, u_b^1, \dots, u_b^{N_f})^t$ (see table 1). The vector: $w = (T_{ex}, \phi_g^1, \dots, \phi_g^{N_f}, Occ, C_{ex})^t \in \mathbb{R}^{n_w}$ represents the exogenous variables (see table 1). $\phi^N(u_w, u_f)$ designates the normalized heat flux emission of the fan coil unit, it is described in the next section.

According to the notation defined in (2), the simulator form of the system (4) is given by:

$$\mathbf{y}_k := \mathcal{Z}(\mathbf{u}_k, \mathbf{w}_k, x_k) \tag{5}$$

which simply means that the trajectory $\mathbf{y}_k$ is the output trajectory for some given sequences $\mathbf{u}_k$ and $\mathbf{w}_k$ starting at instant k from the initial state x_k.

3.2.2. Fan Coil Unit Modelling

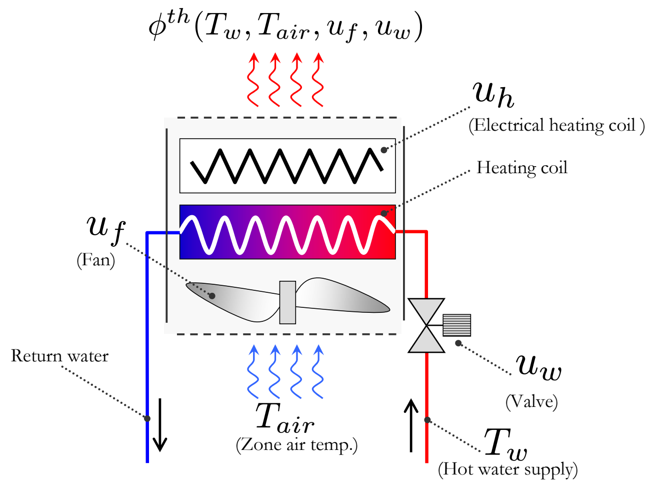

Figure 2: Fan Coil Unit representation. There are two heating coils: an electrical heating coil and a heat exchanger. The fan speed and the valve opening define the heat exchange coeffient ϕ^N.

The figure 2 is a schematic view of a fan coil unit. The system is composed of two heating coils: an electrical heating coil and heat exchanger which enables heat transfer between the hot water and zone air. The amount of heat exchanged between the coil and the zone is given by the following relation:

$$\phi^{th}(u_w, u_f, T_{air}, T_w) = (T_w - T_{air})\cdot\phi^N(u_w, u_f) \tag{6}$$

which means that the heat extracted from the hot water and injected in the zone $\phi^{th}(\cdot)$ is indirectly controlled by the position of the actuators u_w and u_f via the normalized heat transfer function $\phi^N(u_w, u_f)$ and the difference between the hot water temperature and the temperature of the zone. An example of such characteristic function (for a given FCU) is depicted on figure (3). The characteristic function $\phi^N(u_w, u_f)$ has been obtained from the building simulation tool SIMBAD and has been built-up from the physical characteristics of the FCU. Several modelling hypothesis has been assumed in order to derive such characteristic, one can refer to [24, 25] for more details concerning these aspects.

It is easy to see that one can write the system (4) the following form thanks to the introduction of the new control vector $v = [\phi^N(u_w, u_f), u_w, u_f, u_h, u_v, u_l, u_b^1, \dots, u_b^{N_f}]^t$

$$\left\{\begin{array}{lll} x^+ & = & A\cdot x + \Big[B(y,w)\Big]\cdot v + F\cdot w \\ y & = & C\cdot x + D(w)\cdot v \end{array}\right. \tag{7}$$

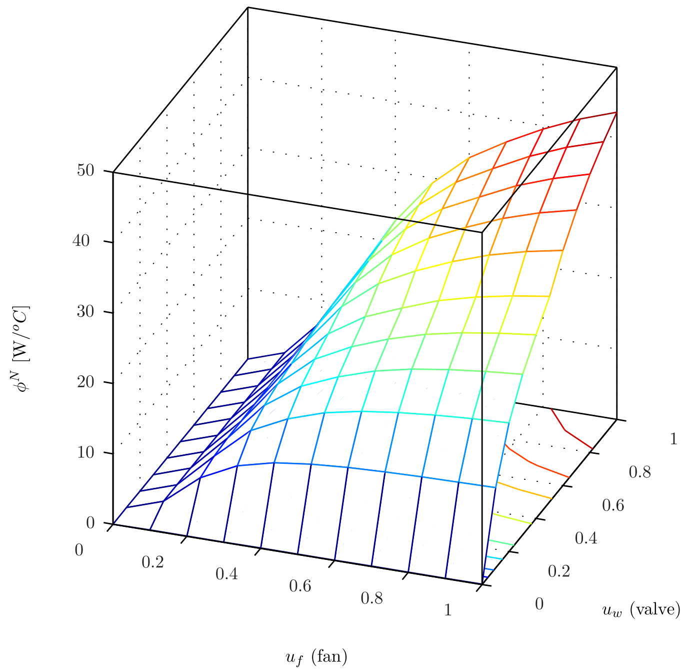

Figure 3: Normalized power flux function ϕ^N.

Then the profile of instantaneous power consumption of the zone (resulting from all the actuators) is given by:

$$\mathbf{p}_k = \mathcal{A}(\mathbf{y}_k, \mathbf{w}_k) \cdot \mathbf{v}_k \tag{8}$$

with:

$$\mathcal{A}(\mathbf{y}_k, \mathbf{w}_k) := \dots$$
$$\mathbf{diag}\{\alpha(y_k, w_k), \dots, \alpha(y_{k+N-1}, w_{k+N-1})\} \in \mathbb{R}^{2N \times N \cdot n_v} \tag{9}$$

We finally add saturations on inputs, for convenience we consider here normalized input vector i.e: $u_k \in [0, 1]^{n_u}$), the NMPC-related optimization problem at instant k becomes:

$$\underset{\mathbf{0} \le \mathbf{u} \le \mathbf{1}}{\textbf{Minimize}} \quad J_k := J^E(\mathbf{\Gamma}, \mathbf{p}) + J^C(\mathbf{y}) \tag{10}$$

where:

- $J^E(\mathbf{\Gamma}_k, \mathbf{p}_k) = [\mathbf{\Gamma}_k^t \cdot \mathcal{A}(\mathbf{y}_k, \mathbf{w}_k)]^t \mathbf{v}_k$ corresponds to the integral energy criterion over the horizon. It depends on the consumed power profile $\mathbf{p}$ and the utility cost $\mathbf{\Gamma}$. Note that $J^E(\cdot)$ is affine in $\mathbf{v}_k$,
- $J^C(\mathbf{y})$ is the discomfort criterion and depends only the outputs $\mathbf{y}$. It is parametrized via the two positive scalars ρ_0, ρ_1 and $\delta_y \in \mathbb{R}^{n_y \cdot N}$, see figure 4.

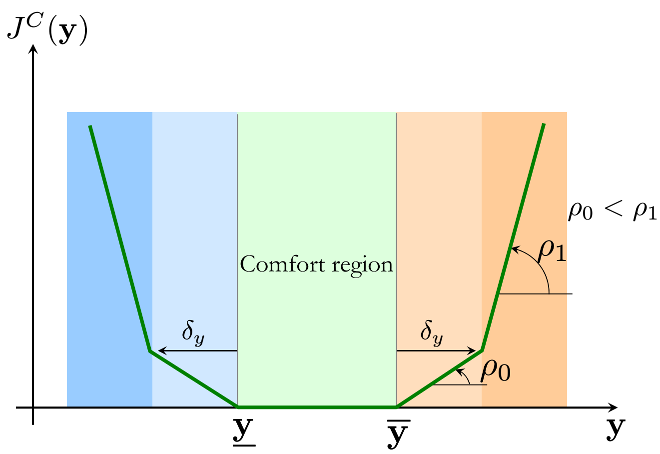

Figure 4: Discomfort function: The discomfort function is paramterized by the ρ_0, ρ_1 and δ_y which represents a reasonable bound violation.

Note that:

- The system described by (4) (or its equivalent (7)) is a nonlinear system,
- The matrices $[B(y, w)]$ and $D(w)$ are affine in their arguments. Moreover the respective columns of $B(y, w)$ and $D(w)$ corresponding to u_w and u_f are equal to zero.

3.3. The Control Problem

In this section, the cost function $J_k(\cdot)$ as well as the zone operational constraints are described.
The knowledge of the following quantities is assumed :

- The current state x_k of the system model (obtained via classical dynamic observer)
- $\mathbf{\Gamma}$ is the predicted utility cost corresponding to each power source,
- The prediction of the exogenous inputs profile $\mathbf{w}_k$
- The comfort related bounds profiles $\underline{\mathbf{y}}_k$ and $\overline{\mathbf{y}}_k$ which are implicitly given by the prediction on occupancy $\mathbf{Occ}_k$

We also assume that the power consumption of electrical equipments is linear in u. However, the amount of heat exchanged between the hot water source and the zone air depends not only on the position of the actuators u_f and u_w, but also on the difference between the temperature of inlet water and zone air (6). hence: $p_k = \alpha(y_k, w_k) \cdot v_k \in \mathbb{R}^2$ is the instantaneous power consumption of the zone, where the vector matrix $\alpha \in \mathbb{R}^{2 \times n_u}$ gathers the marginal power consumption of all equipments.

It is easy to see that the optimization problem (10) can be putted on the following form:

$$\text{NLP}_k\text{: } \underset{\mathbf{v}_k, \delta_1, \delta_2}{\textbf{Minimize}} \; J(\mathbf{v}_k, \mathbf{y}_k, \mathbf{w}_k) \tag{11a}$$

Subject To :

$$[\Phi(\mathbf{y}_k, \mathbf{w}_k)]\mathbf{v}_k + \delta_1^- + \delta_2^- \ge \underline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{11b}$$

$$[\Phi(\mathbf{y}_k, \mathbf{w}_k)]\mathbf{v}_k - \delta_1^+ - \delta_2^+ \le \overline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{11c}$$

$$\mathbf{0} \le \mathbf{v}_k \le \overline{\mathbf{v}} \tag{11d}$$

$$\delta_1 \ge \mathbf{0}\,,\; \mathbf{0} \le \delta_2 \le \begin{bmatrix} \delta_y \\ \delta_y \end{bmatrix} \tag{11e}$$

Where:

- $\delta_1 = \begin{bmatrix} \delta_1^+ \\ \delta_1^- \end{bmatrix} \in \mathbb{R}^{2 \cdot N \cdot n_y}, \delta_2 = \begin{bmatrix} \delta_2^+ \\ \delta_2^- \end{bmatrix} \in \mathbb{R}^{2 \cdot N \cdot n_y}$ are variables introduced in order to describe the function $J^C(\mathbf{y})$,
- The notation **1** indicates a column vector of ones with appropriate size,
- $\Phi(\mathbf{y}_k, \mathbf{w}_k)$,Ψ and Ξ are defined in Appendix A,
- $\bar{\mathbf{v}} = [\underbrace{[1, 1, \infty, 1, \ldots, 1]^t, \ldots, [1, 1, \infty, 1, \ldots, 1]^t}_{N \text{ times}}]^t$ represents the saturation on the control variables v (note that the only unsaturated variable is ϕ_N, since it is naturally limited given that the $\phi_N = \phi_N(u_w, u_f)$ must be satisfied,

The presence of the product terms between the variables y and u and the nonlinear equations $\phi^N(\cdot)$ which is implicitly involved in the system dynamical representation make the problem (11) a non-convex optimization problem which is generally hard to solve. We show in this section how this problem can be efficiently solved using a simple iterative procedure based on the resolution of a sequence of linear programming problems.

3.4. *solving the optimization problem*

In this section, the optimization algorithm used in solving the problem (11) is presented. In order to be able to solve efficiently the problem (11), two main steps are proposed:

1. Piece-wise linear approximation of the nonlinear relation, $\phi^N(\cdot)$. This step is performed off-line,
2. Resolution of resulting optimization problem using a sequence of linear programming problems.

3.4.1. *Piece-wise linear approximation*

The key idea in this part is to approximate the nonlinear function $\phi^N(\cdot)$ by a piece-wise affine approximation noted $\hat{\phi}^N(\cdot)$. The interest of this kind of approximations for our problem will appear in next part. In order to be able to describe approximately $\phi^N(\cdot)$, n_a regions $\mathbf{D}_i$, $i = 1, \ldots, n_a$ in which the function $\phi^N(\cdot)$ can be approximated by a linear function are introduced, Namely:

$$\hat{\phi}^N = \begin{cases} a_1 u_w + b_1 u_f + c_1 & \textbf{if: } (u_f, u_w) \in \mathbf{D}_1 \\ \vdots & \\ a_{n_a} u_w + b_{n_a} u_f + c_{n_a} & \textbf{if: } (u_f, u_w) \in \mathbf{D}_{n_a} \end{cases} \tag{12}$$

The approximation $\hat{\phi}^N(\cdot)$ as well as the regions $\mathbf{D_i}$, $i = 1, \ldots, n_a$ are depicted on figure 5.

Note that the approximation $\hat{\phi}^N(\cdot)$ is strictly concave. Therefore one can deduce that:

$$\hat{\phi}^N(u_w, u_f) = \underset{i=1,\ldots,n_a}{\textbf{Inf}} [a_i, b_i] \cdot [u_w, u_f]^t + c_i \tag{13}$$

Moreover, the subgarph of $\hat{\phi}^N(\cdot)$ -which is the region under the surface defined by (13)- is given by the following set on linear inequalities:

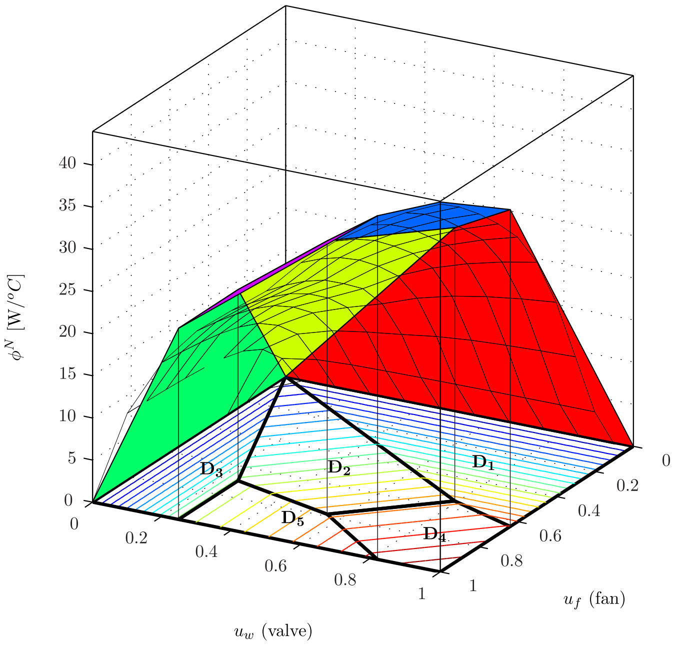

Figure 5: Piece-wise linear approximation of ϕ^N with $n_a = 5$ regions. Notice the approximation is strictly concave.

$$\begin{bmatrix} 1 & -a_1 & -b_1 \\ 1 & -a_2 & -b_2 \\ \vdots & \vdots & \vdots \\ 1 & -a_{n_a} & -b_{n_a} \end{bmatrix} \cdot \begin{bmatrix} \hat{\phi}^N \\ u_w \\ u_f \end{bmatrix} \leq \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n_a} \end{bmatrix} \tag{14}$$

In addition, one can define $S \in \mathbb{R}^{n_a \times n_u}$ and $h \in \mathbb{R}^{n_a \times 1}$ such that:

$$S \cdot v \leq h \tag{15}$$

Where:

$$S := \begin{bmatrix} 1 & -a_1 & -b_1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & 0 & \ldots & 0 \\ 1 & -a_{n_a} & -b_{n_a} & 0 & \ldots & 0 \end{bmatrix}, \; h := \begin{bmatrix} c_1 \\ \vdots \\ c_{n_a} \end{bmatrix} \tag{16}$$

3.4.2. *Fixed-point algorithm*

In this part, a brief overview of the optimization algorithm is provided. The interested reader may refer to [16] for more details concerning this technique. The basic idea of the proposed fixed-point algorithm is to successively approximate the nonlinear inequalities (11b)-(11c) and the cost (11a) around a candidate output trajectory $\mathbf{y}_k^{(s)}$, where $s = 0, \ldots, s_{max}$ is the iteration counter. Hence at iteration s, one has to solve the following mixed integer linear programming problem:

$$\mathrm{MILP}_k^{(s)}: \underset{\mathbf{v}_k,\delta_1,\delta_2}{\textbf{Minimize}}\ J(\mathbf{v}_k,\mathbf{y}_k^{(s)},\mathbf{w}_k) \tag{17a}$$

Subject To :

$$[\Phi(\mathbf{y}_k^{(s)},\mathbf{w}_k)]\mathbf{v}_k + \delta_1^- + \delta_2^- \geq \underline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{17b}$$

$$[\Phi(\mathbf{y}_k^{(s)},\mathbf{w}_k)]\mathbf{v}_k - \delta_1^+ - \delta_2^+ \leq \bar{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{17c}$$

$$0 \leq \mathbf{u}_k \leq 1 \tag{17d}$$

$$\hat{\phi}^N(j) = \underset{i=1,\dots,n_a}{\textbf{Inf}}\ [a_i, b_i]\cdot[\mathbf{u_w}(j),\mathbf{u_f}(j)]^t + c_i, \dots$$

$$j = 0,\dots,N-1 \tag{17e}$$

$$\delta_1 \geq \mathbf{0}\ ,\ \mathbf{0} \leq \delta_2 \leq \begin{bmatrix}\delta_y \\ \delta_y\end{bmatrix} \tag{17f}$$

The optimal input profile $\mathbf{v}_k^{(s)}$ obtained from the resolution of (17) is than injected in the nonlinear system (5) in order to update the current candidate output profile $\mathbf{y}_k^{(s+1)}$, namely:

$$\mathbf{y}_k^{(s+1)} = \mathcal{Z}(\mathbf{u}_k^{(s)},\mathbf{w}_k,x_k) \tag{18}$$

Moreover, given that: $\hat{\phi}^N(\cdot)$ is concave and variables u_f, u_v are only linked to $\hat{\phi}^N(\cdot)$. It can be easily shown that the problem (17) is equivalent to the following linear programming problem, in which (17e) has been replaced by the subgraph of $\hat{\phi}^N$ (19e):

$$\mathrm{LP}_k^{(s)}: \underset{\mathbf{v}_k,\delta_1,\delta_2}{\textbf{Minimize}}\ J(\mathbf{v}_k,\mathbf{y}_k^{(s)},\mathbf{w}_k) \tag{19a}$$

Subject To :

$$[\Phi(\mathbf{y}_k^{(s)},\mathbf{w}_k)]\mathbf{v}_k + \delta_1^- + \delta_2^- \geq \underline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{19b}$$

$$[\Phi(\mathbf{y}_k^{(s)},\mathbf{w}_k)]\mathbf{v}_k - \delta_1^+ - \delta_2^+ \leq \bar{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \tag{19c}$$

$$0 \leq \mathbf{u}_k \leq 1 \tag{19d}$$

$$\mathbf{S}\cdot\mathbf{v}_k \leq \mathbf{h} \tag{19e}$$

$$\delta_1 \geq \mathbf{0}\ ,\ \mathbf{0} \leq \delta_2 \leq \begin{bmatrix}\delta_y \\ \delta_y\end{bmatrix} \tag{19f}$$

where:

$$\mathbf{S} := \mathbf{diag}\{\underbrace{S,\dots,S}_{N\text{ times}}\} \quad \text{and} \quad \mathbf{h} := [\underbrace{h^t,\dots,h^t}_{N\text{ times}}]^t \tag{20}$$

which can be written on the following compact form :

$$\mathrm{LP}_k^{(s)}: \underset{\underline{\mathbf{z}}\leq\mathbf{z}\leq\bar{\mathbf{z}}}{\textbf{Minimize}}\quad \mathbf{L}_k^{(s)}\cdot\mathbf{z}_k \quad \text{s.t.}: \ \mathbf{A}_k^{(s)}\cdot\mathbf{z}_k \leq \mathbf{b}_k \tag{21}$$

Where: the involved matrices: $\mathbf{L}_k^{(s)}, \mathbf{A}_k^{(s)}, \mathbf{b}_k, \underline{\mathbf{z}}, \bar{\mathbf{z}}$ are defined in Appendix B.

Finally, the fixed-point algorithm is the following:

Algorithm 1 Fixed-point algorithm with trust region

1: $s \leftarrow 1$
2: $\mathbf{u}_k^{(0)} \leftarrow [u_{k-1}^\star(1)^t,\dots,u_{k-1}^\star(N-1)^t, u_{k-1}^\star(N-1)^t]^t$
3: $\mathbf{y}_k^{(1)} \leftarrow \mathcal{Z}(\mathbf{u}_k^{(0)},\mathbf{w}_k,x_k)$
4: $e^{(0)} \leftarrow \infty$
5: **while** $e^{(s-1)} \geq \eta$ **do**
6: $\quad\mathbf{u}_k^{(s)} \leftarrow \mathrm{LP}_k^{(s)}$
7: $\quad\mathbf{y}_k^{(s+1)} \leftarrow \mathcal{Z}(\mathbf{u}_k^{(s)},\mathbf{w}_k,x_k)$
8: $\quad e^{(s)} \leftarrow \mathbf{Max}(\|\mathbf{y}_k^{(s+1)}-\mathbf{y}_k^{(s)}\|_\infty, \|\mathbf{u}_k^{(s+1)}-\mathbf{u}_k^{(s)}\|_\infty)$
9: $\quad s \leftarrow s+1$
10: **end while**
11: $\mathbf{u}_k^\star \leftarrow \mathbf{u}_k^{(s-1)}$

where: $\eta > 0$ is a small threshold. The step (2:) of the algorithm is the warm-start strategy, it consists of initializing the optimal control profile with the one found at last iteration with the convenient shifting. A convergence analysis as well as a computational study are provided in section 3.6.

3.5. Input Parametrization and Output checking

It is well known that the computational burden due to the resolution of $\mathrm{LP}_k^{(s)}(\mathbf{y}_k)$ is strongly linked to the number of constraints and decisions variables involved in. Since the problem (21) is high dimensional, it is important to reduce its size. In order to reduce the number of decision variables on inputs, a common strategy consists of searching the optimal control sequence in the set of N_u^{par} piece-wise constant inputs. Moreover, the output profile is undersampled. This means that the predicted output profile $\mathbf{y}_k$ is checked each N_y^{par} samples, see figure 6. The parameter N_s is the refreshing period, it means that the control is updated each N_s time steps. Notice that N_s is in general smaller than N_u^{par}, therefore let us highlight that N_u^{par} piece-wise constant profile does not mean that closed loop control profile is constant N_u^{par}-Piece wise-constant. Actually the input profile is constant per period of N_s, see figure 6.

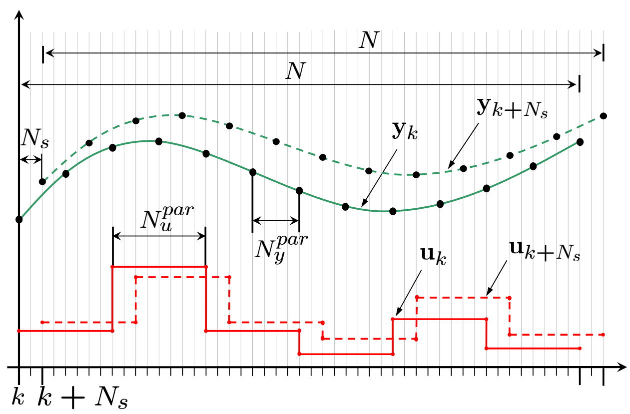

Figure 6: Piece-wise constant parametrization of the predicted control profile and undersampling of the optimal predicted output.

3.6. Convergence analysis and computation time

Even if no formal proof is provided of the convergence, we perform in this part a set of simulations. The figure 7 shows the evolution of error $e(s)$ starting from 100 random initial guesses $\mathbf{u}_k^{(0)}$. One clearly sees that $e(s)$ decreases very quickly, even when starting from very unrealistic initial guesses. It important to remind that a warm start (starting from last solution with convenient shifting) is crucial: using a reasonable tolerance η, convergence is achieved generally in one or two iterations.

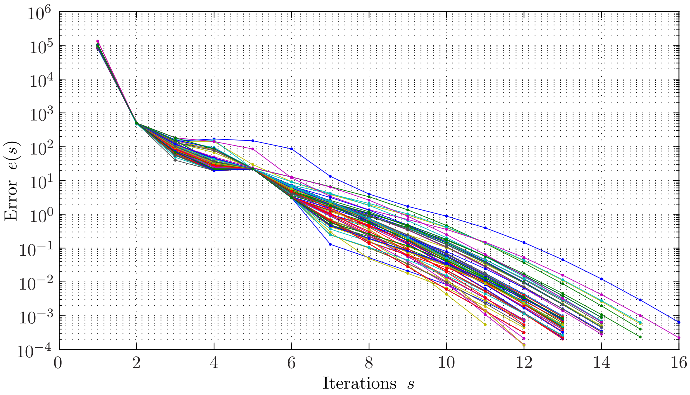

Figure 7: Error $e(s)$ convergence for 100 randomly generated initial guesses $\mathbf{y}_k^{(0)}$. The threshold parameter $\eta = 10^{-4}$. One can see that even when starting from quite unrealistic profiles, the algorithm always converges. Moreover, the error decreases monotonously.

The computation time is extremely low (an average of 10 [ms]) comparing the system dynamics. One can also mention that most the time is spent in problem preparation (i.e: computation of Φ), the figure 8 shows the repartition of computation time. The periodicity in the computation time is due to the fact that the more important is the number of constraints on outputs which are linked to predicted occupancy of the zone, the more computational effort is needed to built-up the matrix $\Phi\cdot$ and to solve the problem (21). Moreover, the only rows of $\Phi\cdot$ that are computed are those corresponding to presence of occupants. This explains the periodicity of the computation time (the flat parts correspond to week-ends).

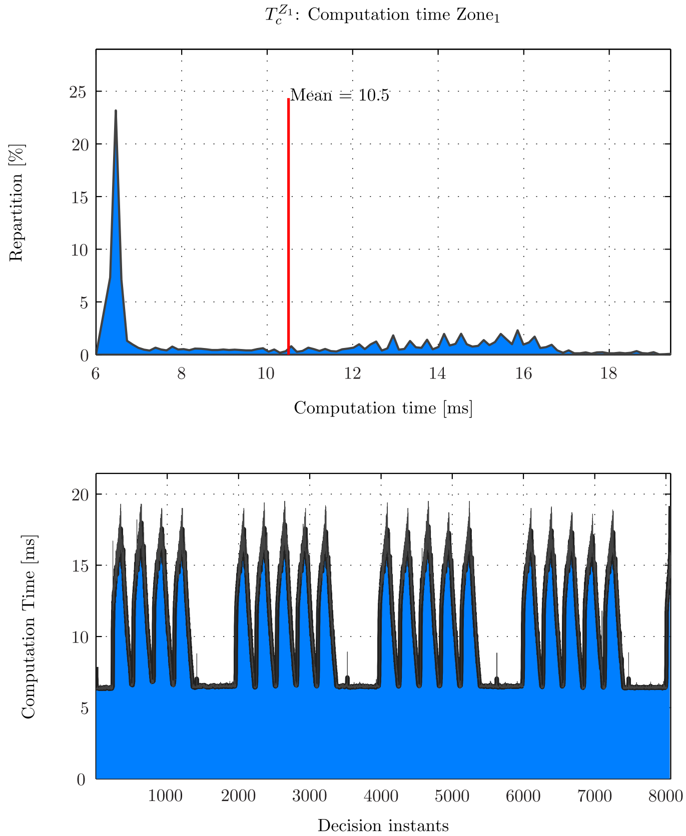

Figure 8: Computation time repartition for $N = 720$, $N_u^{par} = 20$, $N_y^{par} = 20$. These results are obtained on an Intel® Xeon® @ 2.67 GHz, 3.48 Go RAM. ILOG CPLEX 12.1 is used to solve the linear programming problems.

4. Simulation results

In this section, we provide some simulation results in order to assess the proposed NMPC controller. The figure 9 shows simulation results in a typical winter day. The

5. Conclusion

Appendix A. Prediction matrices

$$\Psi := \begin{bmatrix} C \\ CA \\ \cdot \\ CA^{(N-1)} \end{bmatrix}, \Phi(\cdot) = \begin{bmatrix} D_0 & 0 & \cdot & 0 \\ CB_0 & D_1 & \cdot & \cdot \\ CAB_0 & CB_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ CA^{N-2}B_0 & CA^{N-3}B_1 & \cdot & D_{N-1} \end{bmatrix} \quad \text{(A.1)}$$

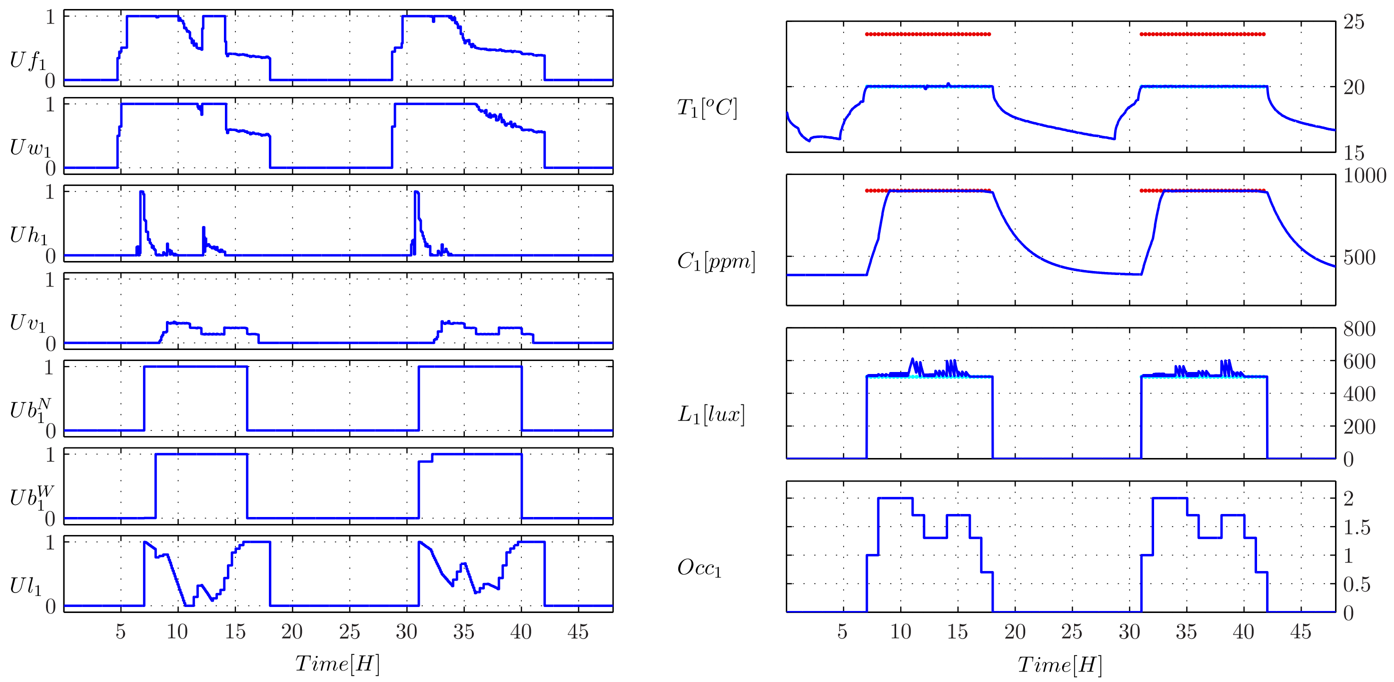

Figure 9: Simulation in winter with FCU

With: $B_i = [B(y_i, w_i)]$, $D_i = [D(w_i)]$, $\quad \forall i \in \{0, \ldots, N-1\}$

$$\Xi = \begin{bmatrix} F & 0 & \cdot & 0 \\ CG & F & \cdot & \cdot \\ CAG & CG & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ CA^{N-2}G & CA^{N-3}G & \cdot & F \end{bmatrix} \tag{A.2}$$

Appendix B. Linear programming problem matrices

$$\mathbf{L}_k^{(s)} := [[\mathbf{\Gamma}_k^t \cdot \mathcal{A}(\mathbf{y}_k^{(s)}, \mathbf{w}_k)]^t, \rho_0 \mathbf{1}^t_{2N\cdot n_y}, \rho_1 \mathbf{1}^t_{2N\cdot n_y}] \tag{B.1a}$$

$$\mathbf{A}_k^{(s)} := \begin{bmatrix} [\Phi(\mathbf{y}_k^{(s)}, \mathbf{w}_k)] & -\mathbf{I} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ [\Phi(\mathbf{y}_k^{(s)}, \mathbf{w}_k)] & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} \\ \mathbf{S} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{B.1b}$$

$$\mathbf{b} := \begin{bmatrix} \underline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \\ \overline{\mathbf{y}}_k - \Psi x_k - \Xi \mathbf{w}_k \\ \mathbf{v}_k \end{bmatrix} \tag{B.1c}$$

$$\overline{\mathbf{z}} := \begin{bmatrix} \overline{\mathbf{v}} \\ \delta_y \\ \delta_y \\ \infty_{2N\cdot n_y} \end{bmatrix}, \ \underline{\mathbf{z}} := \mathbf{0} \tag{B.1d}$$

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