Distributed constrained Model Predictive Control based on bundle method for building energy management

Mohamed Yacine Lamoudi , Mazen Alamir and Patrick Béguery

Abstract— **This paper presents a distributed Model Predictive Control framework based on a primal decomposition and a bundle method to control the indoor environmental conditions in a multisource/multizone building. The control aims to minimize the total energy cost under restrictions on global power consumption and local constraints on comfort and saturations on actuators. Moreover, each power source is supposed to have a time varying tarification. The distributed Model Predictive Control algorithm is based on two layers: a zone layer which is responsible of local zone decisions and a coordination layer that handles decisions that go beyond the scope of the zone. Simulation results are finally provided for a three zones building with a local power production and changing rate grid power. A computational study is also provided, this to attest the effectiveness and the real-time implementability of the proposed control method.**

I. INTRODUCTION

Nowadays,the part of primary energy consumed by buildings is estimated at over 40 % of total primary energy produced worldwide, making building area the largest energy consumer of the world. This explains that decreasing energy consumption in buildings, has become an important component of the CO_2 emission reduction policy and an active research field as witnessed by the important number of publications on this topic [1].

It goes without saying that a complete review of all existent and proposed control strategies lies beyond the scope of the present contribution. Nevertheless, it has to be noticed that Model Predictive Control is identified as a promising methodology in building energy related issues [2].

Unlike usual "rule based strategies" [3], Model Predictive Control is able to give some coherence in the process of decision and to handle economical objectives and multivariable systems, which are very suitable capabilities for the so called **B**uilding **E**nergy **M**anagement **S**ystems.

Several predictive control strategies have been proposed in literature. In [4] a thermal predictive control strategy has been proposed for high inertia buildings to deal with overheat problem. [5] used a predictive controller to manage polygeneration systems. Some very recent experiments [6] showed that Model Predictive Control substantially decreases energy usage comparing to other control strategies. An estimation of

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the potential gain is also given in [2], where a large variety of studies have been conducted to asses the relevance of such strategies in buildings.

In fact, Model Predictive Control is becoming a crucial paradigm for energy efficiency in building control. Nevertheless, one of the major drawbacks in Model Predictive Control lies in the resulting computational burden that may be prohibitive for real-time implementation. This is generally the case in large scale applications. In such situations, the resulting MPC optimization problem is hard to solve given restrictions on computational resources in term of time and memory. On the other hand, it can be unsuitable to centralize the calculation of the optimal solution of a large process in one controller.

This fact is reported in [7], [8], where a distributed framework is adopted to deal with thermal regulation of a multizone building. In [7], a Bender's decomposition strategy is used to take in charge some coupled inputs in presence of multiple heating sources. A multi-agent paradigm is also adopted, in a different way than the previous in [9], [1]. Actually, multi-agent framework is suitable for building where the decomposability of the problem is quite natural.

The main originality of the present contribution lies in proposing a distributed framework to handle global limitations on consumption in presence of more than one power source in multizone building. Moreover, the zones behavior are described by a multiinput/multioutput bilinear state space representations, each zone has to ensure local constraints on actuators and outputs. From our knowledge, the combination of these features has not been proposed previously.

The designed distributed Model Predictive Control is based on two layers: a zone layer and coordination layer. The zone layer gathers local zonal model predictive controllers which are responsible of handling local zonal decision variables to regulate indoor local conditions that are represented by indoor temperature, CO_2 level and illuminance for each zone [10]. The coordination layer ensures that a limitation on global consumption of the building is always respected and enables an optimal dispatch of the limited resources between zones. Moreover, the building is supposed to dispose of a certain number of power sources (e.g: grid + local production) that have time varying tarifications.

The algorithm is based on a primal decomposition and a bundle method for the resolution of the master problem in the coordination layer. It consists of an iterative procedure that ensures always feasible iterates, which is an interesting feature. The sequence of iterates converges toward the optimal solution enabling to recover the optimal solution of

centralized problem.

The paper is organized as follows: section II states the problem. Section III presents the distributed model predictive while giving essential recalls on bundle method. In section IV some simulations are proposed to illustrate the algorithm. Finally, section V gathers conclusions and presents some further issues.

II. PROBLEM FORMULATION

In Model Predictive Control, a model of process is used as well as prediction on disturbances in order to find the best open loop control sequence over the prediction horizon denoted N by minimizing some objective function $J(\cdot)$. Only the first part of the optimal control sequence is injected in the process. This procedure is repeated at the following decision instant based on a new measurement or estimation of the current state and new disturbance prediction. Refer to [11] for a more detailed presentation.

In the sequel, the bolded vectors are used to denote predicted trajectories over the prediction horizon N starting from the current instant t. Namely, if $V \in \mathbb{R}^{n_v}$ then:

$$\mathbf{V}(t) := [V(t|t)^T, \dots, V(t+N-1|t)^T]^T \in \mathbb{R}^{n_v \times N} \quad (1)$$

is the predicted trajectory over the horizon N. For convenience, the current instant t is dropped in the notation $\mathbf{V}(t)$ and it will be simply denoted $\mathbf{V} \equiv \mathbf{V}(t)$ since no ambiguity will result. In the following, the model of the process will be described as well as the MPC related optimization problem.

A. Model description

Consider a building with n_z zones, where $\ell \in Z = \{1, \dots, n_z\}$ is the zone index and let the following nonlinear state space representation describes the dynamical behavior of each zone $\ell \in Z$:

$$x_\ell^+ = A_\ell x_\ell + [B_\ell(y_\ell, w_\ell)]u_\ell + G_\ell w_\ell + H_\ell d_\ell \quad (2a)$$
$$y_\ell = C_\ell x_\ell + [D_\ell(w_\ell)]u_\ell + F_\ell w_\ell + T_\ell d_\ell \quad (2b)$$

Where: $x_\ell \in \mathbb{R}^{n_\ell^x}$, $u_\ell \in \mathbb{R}^{n_\ell^u}$, $w_\ell \in \mathbb{R}^{n_\ell^w}$, $d_\ell \in \mathbb{R}^{n_\ell^d}$, $y_\ell \in \mathbb{R}^{n_\ell^y}$ are respectively state, input, measured disturbance, estimated disturbance and output vector of the zone ℓ.

The key features of the model and some explanations on its form can be found in [10], [12]. However, let us give some necessary elements to understand the sequel:

- The model (2) is a bilinear model since the matrices $[B_\ell(y_\ell, w_\ell)]$ and $[D_\ell(w_\ell)]$ are affine in their arguments.
- Depending on the configuration of each zone ℓ, the vector u_ℓ gathers control on local equipments (HVAC[1], lighting, shading) that may differ between zones. Normalized inputs are considered, i.e $u_\ell \in [0, 1]^{n_\ell^u}$.
- The output vector $y_\ell = [T_\ell^{in}, CO2_\ell^{in}, Lum_\ell^{in}]^T$, includes indoor air temperature, CO_2 level and illuminance in the zone ℓ.
- The measured disturbance vector $w_\ell = [\phi_\ell, T^{out}, OC_\ell, CO2^{out}]^T$, where: ϕ_ℓ are the global

[1]HVAC: Heating, Ventilation and Air Conditioning

irradiance flux on each façade of the zone ℓ, T^{out} is the outdoor temperature, OC_ℓ is the number of occupants in the considered zone and $CO2^{out}$ is the external CO_2 level.
- $d_\ell = [d_\ell^T, d_\ell^C, d_\ell^L]$ is the estimated disturbance vector, it has to be estimated in order to recover any non predicted disturbance that were not taken into account in w_ℓ. The components of d_ℓ respectively represent heat flux, CO_2 production and illuminance flux.

B. Global restrictions on power consumption

It is clear that since no physical coupling between zones is explicitly considered, the zones are totaly independent. Nevertheless, let us mention that a dynamical estimation is used to recover d_ℓ, this enables to take into account indirectly physical interactions between zones as well non modeled disturbances. Suppose now that each zone has access to n_p power sources in the building. These power sources can be of the same nature (e.g electrical: grid + local renewable production) or different (e.g electrical/thermal : grid + boiler). These resources are limited, let us note:

- $\overline{P}^j$ the maximum power profile on the each resource $j \in \mathcal{E} = \{1, \dots, n_p\}$,
- L^j the cost prediction of the resource $j \in \mathcal{E}$.

Moreover, let us define for each zone p_ℓ^j to be the amount of power j consumed by the zone ℓ. Assuming that power consumption is linear with respect to control input, it comes that:

$$\mathbf{u}_\ell = \alpha_\ell \cdot \mathbf{p}_\ell, \quad \ell \in Z, \quad \alpha_\ell \in \mathbb{R}^{N \cdot n_\ell^u \times N \cdot n_p} \quad (3)$$

We are now able to formulate the centralized MPC optimization problem:

$$\underset{\{\mathbf{u}_\ell, \mathbf{p}_\ell, \delta_\ell\}_{\ell \in Z}}{\text{Argmin}} \sum_{\ell \in Z} J_\ell \quad (4a)$$

Subject To : $\forall \ell \in Z$

$$\underline{\mathbf{y}}_\ell - \delta_\ell^- \leq \mathbf{y}_\ell \leq \overline{\mathbf{y}}_\ell + \delta_\ell^+ \quad (4b)$$
$$\mathbf{u}_\ell = \alpha_\ell \cdot \mathbf{p}_\ell \quad (4c)$$
$$\sum_{\ell \in Z} \mathbf{p}_\ell^j \leq \overline{\mathbf{P}}^j, \; \forall j \in \mathcal{E} \quad (4d)$$
$$0 \leq \mathbf{u}_\ell \leq 1, \quad \delta_\ell^+ \geq 0, \quad \delta_\ell^- \geq 0 \quad (4e)$$

Where:
- $J_\ell = \sum_{j \in \mathcal{E}} \langle L^j, \mathbf{p}_\ell^j \rangle + \rho_\ell \cdot \langle \mathbf{1}_\ell, \delta_\ell \rangle$. $\langle \cdot, \cdot \rangle$ is the inner product and $\mathbf{1}_\ell$ is a vector of ones with appropriate size.
- $\underline{\mathbf{y}}_\ell$ and $\overline{\mathbf{y}}_\ell$ are respectively lower and upper output bounds that must be respected in order to ensure occupants comfort. They are obviously linked to the presence of occupants in the zone [10], [8].
- $\delta_\ell = [\delta_\ell^{-T}, \delta_\ell^{+T}]^T$ are some relaxation variables. They ensure that the optimization problem remains always feasible by relaxing the constraints on outputs.
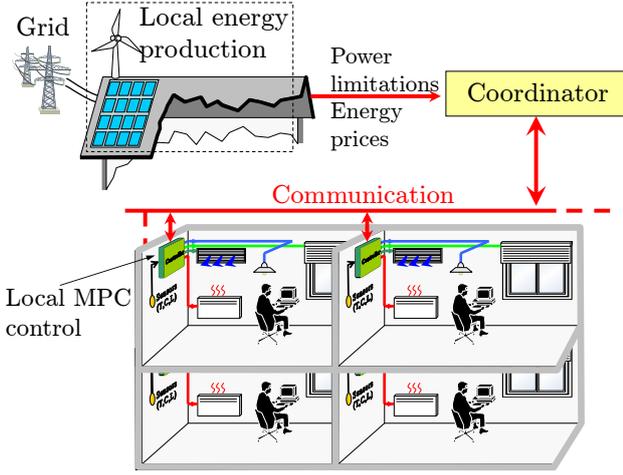- The weight $\rho_\ell > 0$ is adjusted to avoid any unnecessary violation of the constraints on outputs.

Fig. 1. Distributed Model Predictive in a multizone building. The coordinator gathers global information and ensures coordination of local controllers.

It is worth reminding that this optimization problem has to be solved at each decision instant. Since this problem is high dimensional and nonlinear, it can be impossible to solve it in a reasonable amount of time particularly when the number of zones n_z is high. In the next section a distributed predictive control is used to tackle this issue.

III. DISTRIBUTED MODEL PREDICTIVE CONTROL

As mentioned, the centralized optimization problem (4) is very hard to solve or even impossible if n_z is very high as it is the case in office buildings for instance. In addition to the non scalability of the centralized solution, one has to mention that it is generally unsafe to centralize the decision process in the same physical controller because any failure is propagated to the whole system.

To overcome these difficulties, the idea of distributing the computation of the optimal solution among agents appears to be clearly adapted to this situation. The principle of distributed model predictive control [13] is to design local predictive controllers that are responsible of local decision making. The agents have to come with an agreement through iterations in order to recover the solution of the centralized problem or to achieve a relevant solution. The structure of the distributed control and the nature of the exchanged information between agents (and/or a coordinator agent) are the two key points in distributed model predictive control. In this section, the decomposition scheme is presented as well as the coordination mechanism which is performed by a coordinator (fig. 1).

A. Decomposing the problem

The centralized problem (4) is non separable due to the presence of the global constraints (4d). In order to split the problem into subproblems, let us firstly introduce the set of auxiliary variables denoted $\overline{\mathbf{p}} := \{\overline{\mathbf{p}}_\ell^j\}_{(\ell,j) \in Z \times \mathcal{E}}$, such that:

$$\forall (\ell, j) \in Z \times \mathcal{E}: \quad \mathbf{p}_\ell^j \leq \overline{\mathbf{p}}_\ell^j \tag{5}$$

Each vector $\overline{\mathbf{p}}_\ell^j$ represents the resource restriction assigned to the zone $\ell \in Z$ with respect to power $j \in \mathcal{E}$. It goes immediately that the centralized problem (4) admits a feasible solution if $\overline{\mathbf{p}} \in \mathcal{D}$, where:

$$\mathcal{D} := \{\overline{\mathbf{q}} \mid \forall (\ell, j) \in Z \times \mathcal{E}: \sum_{\ell \in Z} \overline{\mathbf{q}}_\ell^j = \overline{\mathbf{P}}^j \wedge \overline{\mathbf{q}}_\ell^j \geq 0\} \tag{6}$$

Clearly, the centralized optimization problem can be separated into n_z subproblems for a fixed admissible $\overline{\mathbf{p}} \in \mathcal{D}$. In fact, each subproblem defines the local MPC_ℓ subproblem:

$$\text{MPC}_\ell(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}}) : \underset{\mathbf{u}_\ell, \mathbf{p}_\ell, \delta_\ell}{\text{Argmin}} \ J_\ell \tag{7a}$$

Subject To :

$$\underline{\mathbf{y}}_\ell - \delta_\ell^- \leq \mathbf{y}_\ell \leq \overline{\mathbf{y}}_\ell + \delta_\ell^+ \tag{7b}$$

$$\mathbf{u}_\ell = \alpha_\ell \cdot \mathbf{p}_\ell \tag{7c}$$

$$\mathbf{p}_\ell^j \leq \overline{\mathbf{p}}_\ell^j, \forall j \in \mathcal{E} \tag{7d}$$

$$0 \leq \mathbf{u}_\ell \leq 1, \quad \delta_\ell^+ \geq 0, \quad \delta_\ell^- \geq 0 \tag{7e}$$

It is easy to see that each zone related MPC_ℓ problem is always feasible thanks to the relaxation variables δ_ℓ and the introduction of the positive auxiliary variables $\overline{\mathbf{p}}$. Actually, this problem is not a linear programming problem because the model (2) is not a linear model. The resolution of MPC_ℓ has been studied in [10] where a fixed-point iterative procedure has been proposed to solve it and will not be detailed in this short communication.

Nevertheless, it remains now to find the optimal resource dispatch $\overline{\mathbf{p}}^\star$. To this end, the master problem (8) has to be solved:

$$\overline{\mathbf{p}}^\star := \underset{\overline{\mathbf{p}} \in \mathcal{D}}{\text{Argmin}} \ J(\overline{\mathbf{p}}) \tag{8}$$

$$\text{Where:} \qquad J(\overline{\mathbf{p}}) = \sum_{\ell \in Z} J_\ell^\star(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}}) \tag{9}$$

And: $J_\ell^\star(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}})$ is the optimal objective function value corresponding to each $\text{MPC}_\ell(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}})$. The coordination mechanism can be summarized as follows:

s1: (**Coordination layer**) Affect for all local MPC_ℓ, $\ell \in Z$ candidate power restrictions $\overline{\mathbf{p}}$.

s2: (**Zone layer**) Solve in parallel the local $\text{MPC}_\ell(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}})$, $\ell \in Z$ and send to the coordinator $J_\ell^\star(\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}})$ and subgradients g.

s3: go to (**s1**) until convergence.

Where: $g = \{g_\ell^j\}_{(\ell,j) \in Z \times \mathcal{E}}$ is the set of subgradients corresponding to $\overline{\mathbf{p}}$. These subgradients are directly given by the dual variables resulting from the resolution of each $\text{MPC}_{\ell \in Z}$. In the next subsection we will see how the coordinator generates candidate power restrictions, namely how to solve the master problem.
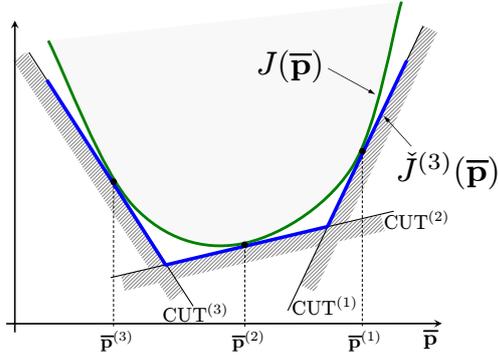
Fig. 2. Representation of $J(\cdot)$ and its piece-wise linear approximation $\check{J}^{(3)}(\cdot)$ at third iteration. The approximate function is a global underestimator if $J(\cdot)$ is convex.

B. Solving the master problem - bundle method

There are several methods that can be employed in order to solve the master problem. However, motivated by the fact that classical subgradient methods may fail to achieve good performances given that $J(\cdot)$ is not differentiable, a bundle method is used. Firstly, let us give a brief recall on the basic idea of bundle method, a more detailed description can be found in [14].

At the kth iteration of the bundle method, k evaluations of $J(\cdot)$ have been performed at k trial points $\overline{\mathbf{p}}^{(1)}, \ldots, \overline{\mathbf{p}}^{(k)}$. Moreover k subgradients $g(\overline{\mathbf{p}}^{(1)}), \cdots, g(\overline{\mathbf{p}}^{(k)})$ (also denoted: $g^{(1)}, \cdots, g^{(k)}$) have been returned from the zone layer $\text{MPC}_{\ell \in Z}$, , their elements correspond to the elements of $\overline{\mathbf{p}}$. The information on the last k iterations is stored in a bundle denoted $\mathfrak{B}^{(k)}$:

$$\mathfrak{B}^{(k)} := \{\overline{\mathbf{p}}^{(i)}, J(\overline{\mathbf{p}}^{(i)}), g(\overline{\mathbf{p}}^{(i)})\}_{i=1,\ldots,k} \quad (10)$$

Based on the bundle $\mathfrak{B}^{(k)}$, the so called cutting plane approximation $\check{J}^{(k)}(\cdot)$ is constructed:

$$\check{J}^{(k)}(\overline{\mathbf{p}}) := \underset{i=1,\ldots,k}{\text{Max}} J(\overline{\mathbf{p}}^{(i)}) + \langle g^{(i)}, \overline{\mathbf{p}} - \overline{\mathbf{p}}^{(i)} \rangle \quad (11)$$

Actually, each linear piece i defines a half space $\text{CUT}^{(i)}(\overline{\mathbf{p}}) := J(\overline{\mathbf{p}}^{(i)}) + \langle g^{(i)}, \overline{\mathbf{p}} - \overline{\mathbf{p}}^{(i)} \rangle - J(\overline{\mathbf{p}}) \leq 0$ as depicted in figure 2.

Given $\check{J}^{(k)}(\cdot)$ at iteration k, one would simply use its minimizer as the next iterate. However, this may lead to some instability because $\check{J}^{(k)}(\cdot)$ can be a poor approximation of $J(\cdot)$ particularly in the first iterations, when only few linear-pieces are available. Hence, instead of minimizing $\check{J}^{(k)}(\cdot)$, the following Quadratic Programming problem is considered:

$$\overline{\mathbf{p}}^{(k+1)} = \underset{\overline{\mathbf{p}} \in \mathcal{D}}{\text{Argmin}} \, \check{J}^{(k)}(\overline{\mathbf{p}}) + \mu_k \|\overline{\mathbf{p}} - \overline{\mathbf{p}}_C^{(k)}\|_2 \quad (12)$$

Where the so called proximal term $\|\overline{\mathbf{p}} - \overline{\mathbf{p}}_C^{(k)}\|_2$ is introduced in order to discourage any drastic movement from the current best candidate point $\overline{\mathbf{p}}_C^{(k)}$, which is called the central point.
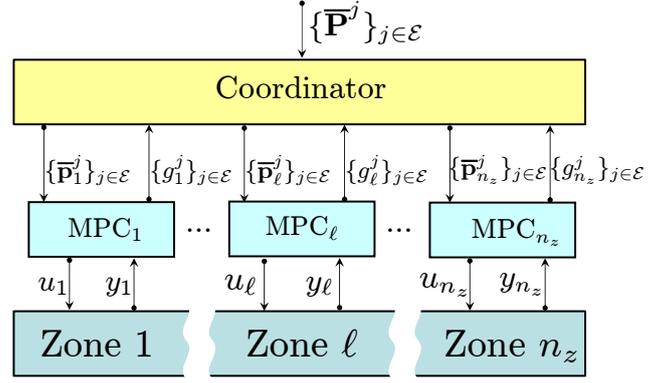

Fig. 3. Illustration of the coordination mechanism. Each $\text{MPC}_{\ell \in Z}$ aim to minimize its local objective (local constraints on actuators + comfort constraints) given some restrictions $\{\overline{\mathbf{p}}_\ell^j\}_{j \in \mathcal{E}}$ on power consumption. Each local $\text{MPC}_{\ell \in Z}$ sends subgradients related to these limitations. The coordinator gathers all the information to solve the master problem by forming $\check{J}(\cdot)$ and resend new resources restriction $\{\overline{\mathbf{p}}_\ell^j\}_{(\ell,j) \in Z \times \mathcal{E}}$, and so on until convergence.

The positive parameter μ_k is the trust region parameter which is adjusted to adequately weight the distance from the current central point. In order to give an update rule of the central point $\overline{\mathbf{p}}_C^{(k)}$ and a stopping test for the algorithm, we need to define the predicted decrease at iteration k:

$$d^{(k)} := J(\overline{\mathbf{p}}_C^{(k)}) - \check{J}^{(k)}(\overline{\mathbf{p}}^{(k+1)}) \geq 0 \quad (13)$$

The point $\overline{\mathbf{p}}_C^{(k)}$ is updated (replaced by $\overline{\mathbf{p}}^{(k+1)}$), if the real decrease is greater than a certain fraction $m \in [0, 1]$ of the predicted decrease namely:

$$\overline{\mathbf{p}}_C^{(k+1)} \leftarrow \overline{\mathbf{p}}^{(k+1)} \quad \textbf{if:} \quad J(\overline{\mathbf{p}}_C^{(k)}) - J(\overline{\mathbf{p}}^{(k+1)}) \geq m \cdot d^{(k)} \quad (14)$$

In this case, the step k is called a serious step, otherwise it is called a null step. However, it is important to notice that in both situations the accuracy of the approximation $\check{J}^{(k)}$ is improved by adding a new element in the bundle:

$$\mathfrak{B}^{(k+1)} = \mathfrak{B}^{(k)} \cup \{\overline{\mathbf{p}}^{(k+1)}, J(\overline{\mathbf{p}}^{(k+1)}), g(\overline{\mathbf{p}}^{(k+1)})\} \quad (15)$$

The algorithm stops when the predicted decrease is lower than a predefined accuracy on the objective function:

$$d^{(k)} \leq \epsilon_J, \quad \epsilon_J > 0 \quad (16)$$

Or if the iteration counter k reaches the maximum number of iterations permitted k_{max}. Finally, it has to be noticed that a FIFO storage strategy is employed to manage the size of the bundle given that has to be limited due to memory limitations to $n_\mathcal{B}$. This means that the oldest element of the bundle is dropped and replaced by the newer one.

In closed loop simulations, the bundle is cleared at the beginning of each negotiation phase. However, the procedure is

initialized by the last best solution known with the convenient shifting (warm start).

Under assumption of convexity on $J(\cdot)$, iterates of the bundle algorithm (summarized on fig. 3) converge toward the optimal solution of the centralized optimization problem (4), [14].

Let us also emphasize on the fact that all iterates are feasible in the sens of respecting all global and also local constraints. This is ensured by forcing iterates to belong to the domain $\mathcal{D}$, this feature is very interesting since the algorithm can be stopped, if necessary, at any iteration.

IV. SIMULATION RESULTS

A. Simulation results

The following scenario is assumed:

- The number of zones $n_z = 3$. The zones models are derived from SIMBAD simulation tool [15]. They have different dynamical behaviors and equipments consumptions. However their occupation schedules are identical. Each zone disposes of an electrical heating, a ventilation system as well as a lighting device and two shutters (fig. 4).

- The number of power sources is $n_p = 2$. The zones have access to electrical power via grid and local electrical production via solar and eolian production. The power consumed from grid is supposed to be limited at $\overline{\mathbf{P}}_1(\cdot) = 10[kW]$ (the zones are able to consume $18.45[kW]$). The grid power price changes over time: it is two times higher during the period $[6\,\text{a.m}, 10\,\text{p.m}]$. The profile of local production $\overline{\mathbf{P}}_2$ is depicted on fig 5, its price is zero.

- The prediction horizon is $N = 720[min.] = 12[hour]$. The update period (control horizon) is fixed to $20[min.]$.

- The meteorological data in our experiments corresponds to Paris meteorological weather station of January 1st (winter season) provided with SIMBAD.

We remind that the objective of each zone is to keep its outputs (air temperature, CO_2 level, illuminance) within a predefined range of values $\overline{\mathbf{y}}_\ell, \underline{\mathbf{y}}_\ell, \ell \in Z$ beside respecting local constraints ($\forall \ell \in Z : u_\ell \in [0,1]^5$). The bounds on outputs obviously depend on the occupation of the zone as depicted on fig. 4 (lines in bold cyan and bold red).

The bundle algorithm related tuning parameters are:

- The accuracy on objective function $\epsilon_J = 10^{-3}$,
- The maximum number of iteration and the bundle size, $k_{max} = n_{\mathcal{B}} = 50$,
- The trust region parameter is constant $\mu_k = 10^{-3}$, $k \in \{1, \ldots, k_{max}\}$,
- The ratio $m = 0.1$.

B. Simulation discussion

Figure 4 depicts closed loop profiles of zones variables (inputs and outputs) while figure 5 shows the resulting closed loop power profiles as well as power consumption of the zones. Note that global constraints on power consumption are respected (fig. 5), this is also the case for zones inputs
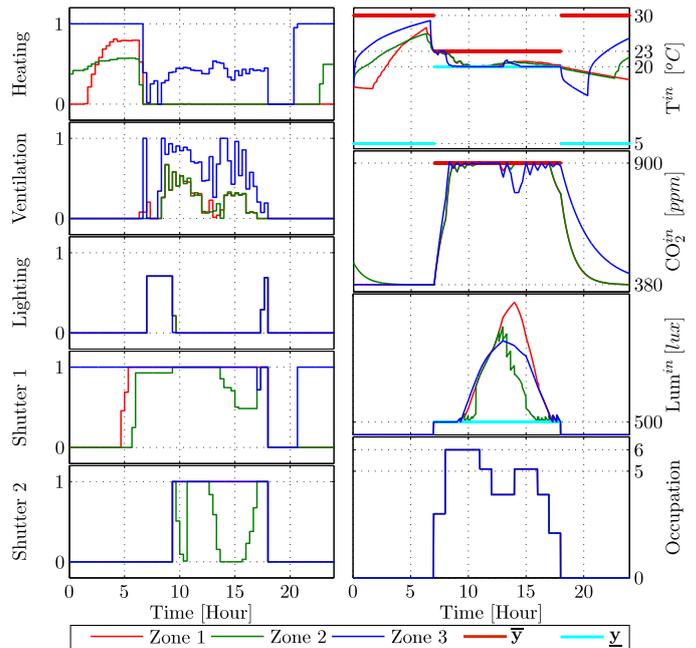

Fig. 4. Closed loop simulation results for $n_z = 3$ zones. The local constraints on comfort (bold cyan and bold red) and on actuators are respected ($u_\ell \in [0,1]^5$). During occupation time the constraints on comfort are more stringent [10].
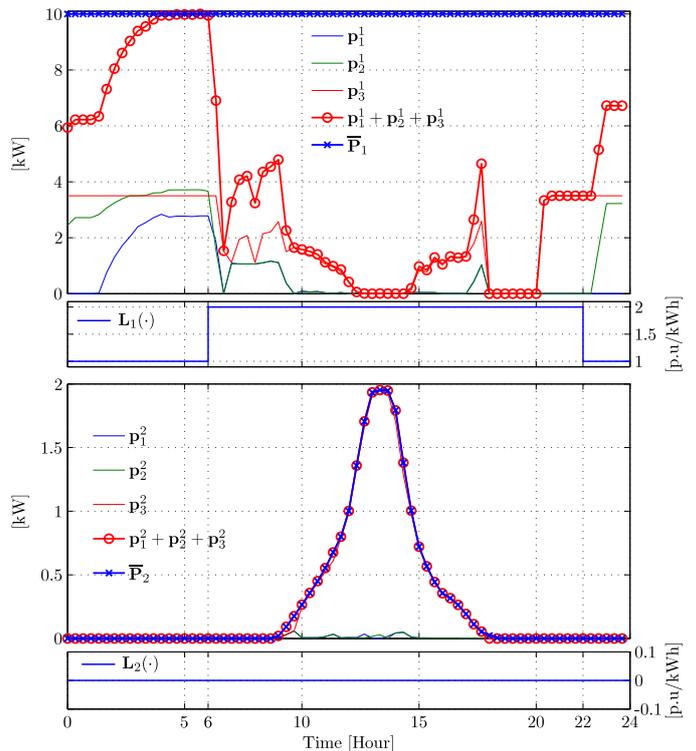

Fig. 5. Global constraints on power consumption. First figure: power consumed from grid, the constraints are respected while anticipating heating when power is cheaper (before 6 a.m). Second Figure: local production power, the constraints are saturated.

saturations (fig. 4). This is not surprising since hard constraints are imposed on these variables. However, it has to be noticed that the anticipative effect of the control strategy enables the zones to take advantage of their inertia in order to store energy (heat) during the period when energy is cheaper (before 6 a.m) in order to minimize the amount of energy bought from grid during the period when energy is more expensive. This is performed without violation of the global of grid power constraint (see temperature profiles and heating control on fig. 4 and grid power consumption fig. 5). It is interesting to remark that the temperature peak observed in $zone_2$ is higher than the ones in the other zones, shortly speaking it means that storing energy in the $zone_2$ is more economically interesting than to perform it in other zones. This illustrates the optimal power dispatch performed by the coordinator, remember that the coordinator doesn't have explicitly access to zones dynamical representations and that zone related information is "summarized" in the bundle. On the other hand, notice that the total amount of local production is consumed. This to reduce the amount of power consumed from grid. Finally, see also that, independently of the control profiles, the constraints on outputs are respected, ensuring comfort for occupants.

C. Brief computational analysis

An important issue concerns the computational time required to achieve convergence. Since the local zone controllers solve their related optimization problems in parallel, the total amount of time $T^{\text{Tot/Iter}}$ spent during one iteration can be stated as:

$$T^{\text{Tot/Iter}} = \underset{\ell \in Z}{\text{Max}}(T_\ell^{\text{Zone/Iter}}) + T^{\text{QP/Iter}} \qquad (17)$$

Where: $T_\ell^{\text{Zone/Iter}}, \ell \in Z$ is the computation time of each zone MPC, this time obviously dependents exclusively on the complexity of the local problems. However, $T^{\text{QP/Iter}}$, which is the amount of time required to solve the master problem, is linked to the number of zones as well as the number of power sources and the bundle iteration counter k. The figure 6 depicts histograms of $T_\ell^{\text{Zone/Iter}}$ and $T^{\text{QP/Iter}}$, it can be seen that $T^{\text{QP/Iter}}$ and $T_\ell^{\text{Zone/Iter}}$ are quite equivalent. This indicates that the computational efficiency of the master problem is crucial since it is far from being negligible comparing to the local MPC's optimization problems as it can be the case with other methods.

Moreover let $T^{tot} := T^{QP} + T^{Zone}$ be the total computation time required for convergence of the bundle algorithm. Where: T^{QP}, T^{Zone} are respectively the total amount of time spent in solving the QP master problem and the amount of time spent in the zone layer. It can be noticed that the maximum computation time T_{Max}^{tot} is approximately $8.5[s]$. This computational time is sufficiently small for our application enabling for instance a refreshing period of $1[min]$. This corresponds to a maximum number of iterations of 29, however the mean number of iterations needed to achieve convergence (in our case study) is 6 (fig. 8).
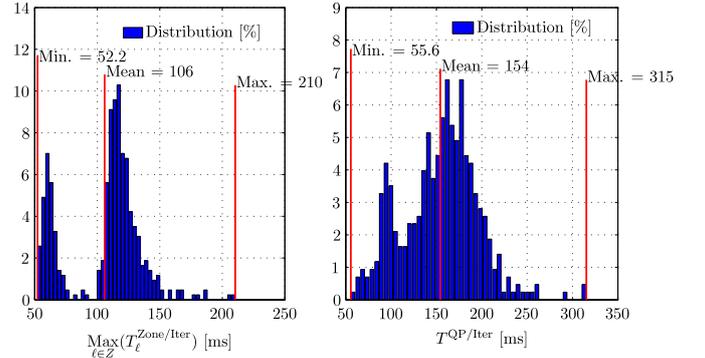

Fig. 6. Computation time distribution of the computation time in the zone layer (left) and master problem (right) per iteration. It can be noticed that they are quite equivalent.
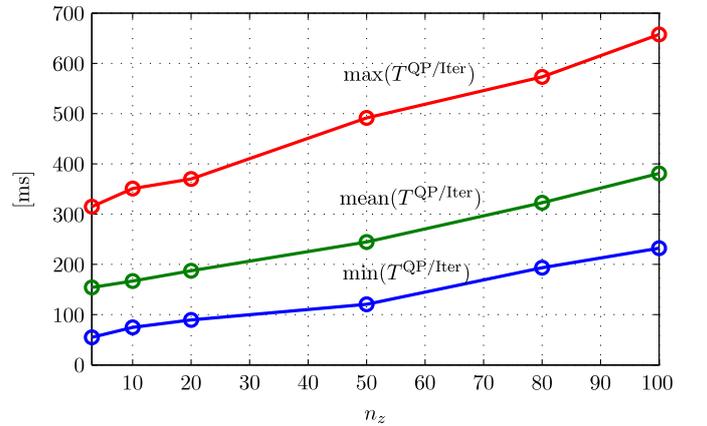

Fig. 7. Computation time of the master problem vs. number of zones n_z. It can be seen that the computational time of the master problem increases slowly whit the number of zones n_z, ensuring the scalability of the algorithm

Let us also evaluate the computational time of the master when the number of zones increases (fig. 7).

These results were obtained on an Intel® Core(TM) $i7$ CPU X920 @ 2.00 GHz, 3.23 Go RAM. ILOG CPLEX 12.1 solver was used.

V. CONCLUSION

In this paper, a distributed model predictive control algorithm has been proposed to handle global limitations on power consumption in a multizone building. The algorithm is based on an iterative procedure that produces feasible iterates. The use of a bundle algorithm to solve the master problem ensures the efficiency of scheme. The proposed scheme has been assessed on a multisource/multizone building, attesting the real time implementability of the proposed algorithm. While in this preliminary study, only the fundamental aspects of the scheme have been exposed, further studies will focus on extending the proposed technique to handle coupled inputs as well as more complex energy systems, this is the case for instance in double flux air handling units. Another issue concerns the resulting QP optimization problem that
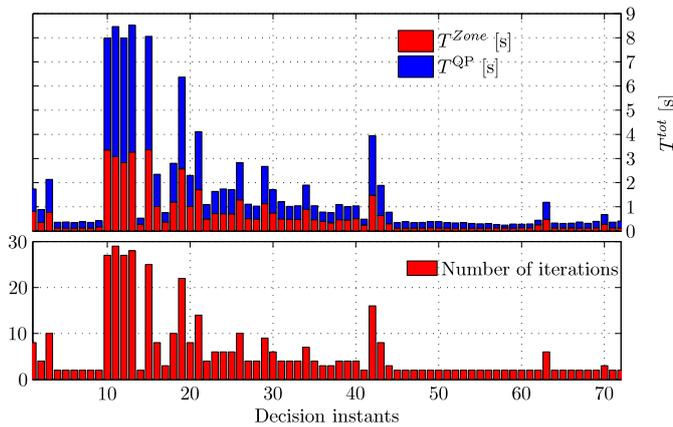
Fig. 8. Total computation time and number of iterations required to converge at decision instant.

may be too large if the number of zones is very high. A disaggregated version of the bundle algorithm, based on forming subsets of zones, may be a judicious solution.

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