Distributed Partially Cooperative NMPC Under Limited Communication and Destabilizing Interconnections

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Abstract: In this paper, a new formulation of distributed and partially cooperative control under limited communication is proposed. Unlike many existing schemes, destabilizing interconnections are considered and partial *load shading*-like decisions may be potentially taken in order to maintain the overall system integrity. The example of power systems black-out can be viewed as the targeted context although the present paper gives the general framework regardless any specific application.

Keywords: NMPC; Networks of Nonlinear Systems; Communication Constraints; Stabilization; Partially Cooperative Systems

1. INTRODUCTION

The problem of controlling a network of interconnected systems is not a new issue: Hierarchical control of large-scale dynamical systems has been a subject of great interest since the 60's: thanks to optimization theory, the so-called multi-level algorithms (or Decomposition-Coordination approaches) have been proposed (Lasdon (1970), M. D. Mesarovic and Takahara (1970)). Most of these algorithms rely on Lagrangian relaxation techniques. As a consequence, many of theses algorithms fail to converge when the existence of saddle-point of the Lagrangian is not ensured (only in the case of strongly convex problems). This is one of the reasons why the application of multi-level approaches have not been extensively used.

However, the development of networked control system and the increasing computational facility together with the fact that these systems are more and more used in a quite critical context raised a new interest in this subject during the few past years (Larsson and Skogestad (2000), Negenborn et al. (2004), Kim and Sugie (2005), Franco et al. (2005), Dunbar (2005), Venkat et al. (2005)).

In particular, solutions based on Model Predictive Control schemes (**MPC**) have been extensively investigated during the last decade in both linear (Kim and Sugie (2005), Venkat et al. (2005)) and nonlinear context (Franco et al. (2005), Dunbar (2005)).

Regardless of the kind of solutions, the researches in this field try to address the following problems :

(1) How to decompose a large scale systems into a set of interconnected subsystems.
(2) How to assign "*local problems*" to local controllers
(3) How to make the local controllers cooperate and communicate in order to yield a good solution of the original problem.

Each of these problems constitutes a field for system theoretical investigation and it seems that there are no generic solutions to any of them.

A deep and detailed analysis of existing works would be clearly beyond the scope of this short contribution. However, the following items may help to position our ideas in the corpus of existing approaches :

- In many approaches, in order to handle the large scale feature, the decomposition of the system is done based on the nonlinearity (Zhu and Henson (2002)). Namely, if no nonlinearities exist, only one system is used and a centralized controller is adopted.

  This clearly does not address the problem at hand because many real-life systems are built up with subsystems having their own local controllers with limited information exchange between subsystems.

- We believe that the large scale character of the problem, while sometimes problematic, is far from being the central problem. Classical control theory already solved problems arising in process industry with thousands of variables and degrees of freedom.

  The key problem lies in the following features that are clearly emphasized for large scale systems.

  · **Limited resources sharing** This constraint means that the constraints on local controllers are not decoupled. The amount of available control at some local controller level heavily depends on the behaviors of the other controllers. This feature is rarely taken into account although it is in the heart of very hot problems such as the "*blackout*" arising in electrical power distribution.

The consequence of this resource limitation is that the level of performance of one subsystem can influence that of its neighbors. In such a situation, a kind of trade-off has to be implicitly implemented in the control structure.

- **Limited communication range** This constraint means that the available information at each local controller level concerns only a subset of the information that describes its own neighbors.
- **Communication constraints** Data communications are subject to various constraints such as limited bandwidth, asynchronism, transmission delays or data loss. These constraints are not often taken into account in the context of distributed MPC.
- **Unstable interconnections between subsystems** In many existing approaches, it is assumed that the coupling flows between subsystems cannot lead to instability. More precisely, when each subsystem uses its already defined controller that is based on its own state, the resulting closed-loop system cannot be destabilized by the interconnection.

  For potentially unstable distributed systems, different priority levels have to be assigned to stability and optimality. This can be done by transient decrease of the subsystems requirements in order to recover the stability. In a second time and using a larger settling time, optimality of performance can be recovered. Very few works take care of such considerations altogether.

Many existing works do not tackle explicitly these features. Indeed, these features make irrelevant the use of iterative schemes in which *centralized* steps are used (Camponogara (2002)) or those where control limitations are decoupled (Dunbar (2005)). They are also incompatible with the assumption of *stable* interconnections between subsystems Venkat et al. (2005) or that of linear models (Kim and Sugie (2005), Venkat et al. (2005)) that do not fit many unstable behaviors that occur often in the nonlinear operation domain.

The present work does not address all of the above problems. However, the proposed control scheme partially answers some of them. More precisely, we consider an interconnection of nonlinear subsystems, each of them already has its own controller and index of performance. The interconnection may be destabilizing. The communication is limited since each subsystem can communicate with its immediate neighbors and with a limited amount of data exchange. Moreover, the control scheme enables the expressions of trade-off and even the possibility for a subsystem to be self-shaded in order to protect the overall network survival.

The paper is organized as follows: First, some definitions and preliminary facts are given in section 2. In particular, the network equations are given (section 2.1), the observer used in each local subsystem in order to reconstruct the relevant part of the interconnection signals is suggested (section 2.2). Moreover, the local control scheme related definitions are introduced together with the local performance indicator functions (section 2.3). The communication rules adopted in the proposed solution is given in section 3. More precisely, the structure of the information exchange and the amount of information that can be shared between neighbors subsystem is rigorously stated. The distributed and partially cooperative control scheme is presented in section 4. More precisely, the concept of available for each subsystem is introduced (section 4.1), the rules governing the definition of the cost functions used in the NMPC scheme are clearly stated (section 4.2) and finally the resulting feedback law at each subsystem level is rigorously defined (4.3). Finally, the paper ends with some concluding remarks and hints for future works.

2. NOTATIONS AND PRELIMINARY FACTS

In this section, some preliminary facts and notations are introduced in order to set properly the interconnection induced control problem.

2.1 Network equations

Assume a network of nonlinear systems $(S_i)_{i=1}^{n_s}$. Suppose that each system S_i is described by:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j\in I_i} G_{(i,j)}(x_i, \xi_i^j) \qquad (1)$$

$$y_i = H_i(x_i, \{\xi_i^j\}_{j\in I_i}) \qquad (2)$$

$$z_i = g_i(x_i) \qquad (3)$$

where $x_i \in \mathbb{R}^{n_i}$ is the state of the system S_i, $y_i \in \mathbb{R}^{n_{y_i}}$ is the vector of measurements available at system S_i, $I_i = \{i_1, \dots, i_k, \dots\}$ is the set of indices of systems $\{S_j\}_{j\in I_i}$ that have influence on S_i. For each $j \in I_i$, $\xi_i^j = h_{(i,j)}(x_j)$ represents a coupling output of system S_j and hence expressed in terms of the state x_j. This output is involved in the expression of the action of S_j on S_i. Finally, $z_i \in \mathbb{R}^{n_{z_i}}$ represents the vector of regulated output of S_i.

Note that the first term in (1) represents the controlled dynamic of S_i if the network is in some nominal design state. The second term expresses all the coupling actions on S_i coming from systems S_j with $j \in I_i$ when the network deviates from the nominal design state.

2.2 Observer at system S_i

It is assumed that each system S_i runs an extended state estimation to produce estimations $\hat{x}_i$ and $\hat{\xi}_i^j$ of x_i and ξ_i^j respectively. This can in particular be done using a Moving Horizon Observer (MHO) scheme based on the assumption of a constant evolution of the unknown coupling quantities ξ_i^j. More precisely, the following problem is solved repeatedly at each sampling period $[t_{p-1}, t_p]$ in order to produce the estimation $\hat{x}_i(t_p)$ and $\hat{\xi}_i^j(t_p)$:

$$\hat{x}_i(t_p) = X_i(t_p, t_{p-N}, \bar{x}_{i,0}, \bar{\xi}_{i,0}^j) \quad (4)$$

$$\hat{\xi}_i^j(t_p) = \bar{\xi}_{i,0}^j \quad (5)$$

$$(\bar{x}_{i,0}, \bar{\xi}_{i,0}^j) \leftarrow \min_{(x_{i,0}, \xi_{i,0}^j)} J(x_{i,0}, \xi_{i,0}^j) := \quad (6)$$

$$\sum_{m=p-N}^{p-1} \|H_i(X_i(t_m, x_{i,0}, \xi_{i,0}^j)) - y(t_m)\|^2 \quad (7)$$

in which $X_i(t_m, x_{i,0}, \xi_{i,0}^j)$ represents the solution of (1) at instant t_m starting from the initial state $x_{i,0}$ at instant t_{p-N} and using the constant value $\xi_{i,0}^j$ over the time interval $[t_{p-N}, t_p]$. Recall that any other dedicated observer, if any, can be used. The moving horizon observer defined by (4)-(7) is proposed here for completeness.

2.3 Nominal controller at system S_i

In this paper, we focus on those aspects that are linked to the interactions and assume that in the absence of these interactions, the control problem at each individual system level is already solved in the nominal context, via some dedicated feedback law $K_i(x_i, z_i^d)$ that depends on the state x_i and the desired output z_i^d. This assumption matches perfectly well many realistic situations where the system S_i's controller design is done for the nominal situation assuming some nominal value of the coupling variables. These values are then considered in the design of the controllers K_i. It is also assumed that this pre-existing controller K_i is such that there is some positive function V_i reflecting the control objective at the system S_i level that satisfies the following inequality:

$$\text{(Nominal)} \quad \frac{dV_i}{dt}\Big|_{\dot{x}_i = F_i(x_i, K_i(x_i, z_i^d))} = -W(x_i, z_i^d) \quad (8)$$

where W_i is a positive function. Finally, we assume that the safe region for system S_i is defined (without loss of generality) by the inequality

$$V_i(x_i) \leq 1$$

Now, according to (1), when S_i is influenced by S_j, the inequality (8) that guarantees the achievement of the control goal for system S_i is disturbed by the additional term:

$$W_i^j(x_i, \xi_i^j) := \left[\frac{\partial V_i}{\partial x_i}(x)\right] \cdot G_{(i,j)}(x_i, \xi_i^j) \quad (9)$$

that may be sufficiently positive to seriously disturb the behavior of S_i. In this case, partial cooperation between S_i and S_j may be necessary. However, in order to have a limited communication solution, we seek a cooperation framework in which neither S_i disposes of the value of $\xi_i^j = h_{(i,j)}(x_j)$ nor S_j knows the value of x_i.

3. THE COMMUNICATION RULES

In the proposed scheme, the following cheap communication rule is adopted:

Hypothesis 1. **[Communication rule 1]**

If S_j *acts on* S_i, then S_i sends the following information to S_j:

$$\mathcal{I}_{i\rightarrow j} := \left\{\hat{\eta}_i = V_i(\hat{x}_i), \left\{W_i^j(\hat{x}_i, z_{ij}^k) =: W_i^j(k)\right\}_{k=1}^{n_{ij}-1}\right\} \quad (10)$$

where $\hat{x}_i$ is the value delivered by the observer used by system S_i. $\{z_{ij}^k\}_{k=1}^{n_{ij}-1}$ is a set of $n_{ij}-1$ pre-defined possible values of the coupling stream ξ_i^j. These quantities can readily be computed by S_i regardless of the present value of ξ_i^j which is assumed to be unknown for S_i. ♭

Note that the above communication rule results in n_{ij} scalars that have to be sent from S_i to S_j regardless the dimension of the state x_i. The first information $\hat{\eta}_i$ informs S_j on the *distance to danger* of S_i. The remaining information enables S_j to *reconstruct* the function $W_i^j(\hat{x}_i, \cdot)$ without the explicit knowledge of $\hat{x}_i$. It goes without saying that the relevance of this reconstruction depends on the number $n_{ij}-1$ of available *interpolation* nodes $\left\{z_{ij}^k, W_i^j(k)\right\}_{k=1}^{n_{ij}-1}$.

In the remainder of the paper, the dynamic reconstructed version of $W_i^j(\hat{x}_i(t), \cdot)$ computed by S_j is denoted by $\hat{W}_i^j(t, \cdot)$. Recall that the second argument of this function is the coupling output $\xi_i^j = h_{(i,j)}(x_j)$ involved in the action of S_j on S_i according to (1).

Note that at some decision instant t_p and using the past information $\{\mathcal{I}_{i\rightarrow j}(t_m)\}_{m=p-N}^{p-1}$ received from S_i, the system S_j can view the past received values $\{\hat{\eta}_i(t_m)\}_{m=p-N}^{p-1}$ as the successive output values of some unknown system with input sequence $\{\hat{W}_i^j(t_m, \xi_i^j(t_m))\}_{m=p-N}^{p-1}$. Consequently, the computation facility at the system S_j level may identify a dynamic prediction model for $\eta_i = V(x_i)$ that takes the following form:

$$[\bar{\eta}_i^j]^+(t_p) = A_i^j \cdot \bar{\eta}_i^j(t_p) + B_i^j \cdot \hat{W}_i^j(t_p, \xi_i^j(t_p)) \quad (11)$$

$$\hat{\eta}_i^j = C_i^j \bar{\eta}_i^j \quad (12)$$

and that delivers as output the prediction $\hat{\eta}_i^j$ of η_i. Note that at the current instant t_p, the value of the internal state of the model (considered by system S_j) relative to its impact on system S_i is given by:

$$\bar{\eta}_i^j(t_p) := \begin{pmatrix} \hat{\eta}_i(t_p) \\ \hat{\eta}_i(t_{p-1}) \\ \vdots \\ \hat{\eta}_i(t_{p-q+1}) \end{pmatrix} \in \mathbb{R}^q \quad ; \quad q < N \quad (13)$$

Note that a standard way to derive such a model is to use the Brunowski canonical form related identification algorithms.

This model can then be used by S_j in order to balance the degree of cooperation it can offer to help system S_i when appropriate and based on its estimation of η_i as it is shown in the next sections.

4. THE DISTRIBUTED PARTIALLY COOPERATIVE NMPC SCHEME

Recall that using an NMPC scheme at the system S_j level amounts to solve at each decision instant t an open-loop optimal control problem associated to some cost function $J(D_j(t), \mathbf{u}_j)$ in which the available data $D_j(t)$ (see section 4.1 for the exact definition of the available data vector D_j) is used as parameter and the sequence of piecewise future control actions $\mathbf{u}_j(\tau)$ for ($\tau \in [t, t+T]$) is the decision variable.

More precisely, using some sampling period $\tau_s > 0$, the control profile over the future prediction horizon $[t, t+T]$ is parameterized using some parametrization map (the prediction horizon length T is assumed to be a multiple of the sampling period τ_s, namely $T = N\tau_s$):

$$\mathcal{U}_{pwc}(\tau, p) := \big(u^{(1)}(p), \dots, u^{(N)}(p)\big) \tag{14}$$

where the control $u^{(k)}(p)$ is applied during the time interval $[t+(k-1)\tau_s, t+k\tau_s]$. By doing so, the cost function J can be viewed as a function of the control parameter vector p, namely $J(D_j(t), p)$.

As it is shown hereafter (see section 4.2), the cost function $J(D_j(t), \mathbf{u_j})$ expresses concerns on the performance of system S_j but also on the viability and even performance of the neighboring systems S_i, $i \in J_j$. The fact that the resulting NMPC is referred to as a distributed scheme comes from the fact that only the neighboring systems S_i, $i \in J_j$ influence the design of the control u_j (defined at system S_j). Moreover, only a partial and a communication cheap part of the information related to these neighboring systems is involved in the computation of the control u_j.

4.1 The available data for system S_j

The key idea in the distributed partially cooperative NMPC proposed in this paper is to provide each system S_j with a *communication-cheap* prediction model of the key quantities $\{\eta_i\}_{i\in J_j}$ where J_j is the set of indices of systems S_i that are influenced by S_j [1]. Such model is given by (11)-(12). Using these models, the system S_j can predict the consequence of any choice of his own control profile $u_j(\cdot)$ on the performance and/or survival [2] of the neighboring systems that are affected by this choice, namely $\{S_i\}_{i\in J_j}$.

More precisely, at some decision instant t_p, given:

- the current estimated state $\hat{x}_j(t_p)$ of S_j delivered its own observer (see section 2.2)
- the present estimated values $\hat{\eta}_i(t_p) = V_i(\hat{x}_i(t_p))$ of systems S_i s.t. $i \in J_j$
- the current internal states $\bar{\eta}_i^j(t_p)$ of the related prediction models (11)-(12) for $i \in J_j$. recall that these values are sent to S_j in accordance with the communication rule (10).
- a candidate future control profile $\mathbf{u_j}$ defined on some prediction interval $[t_p, t_p+T]$

The system S_j can compute the future prediction of its own indicator $\hat{\eta}_j$ based on the prediction of its own state starting from $\hat{x}_j(t_p)$ and using the estimations $\hat{\xi}_j^i(t_p)$ for all systems S_i such that $i \in I_j$. For each control profile $\mathbf{u_j}$, the system equations (1) can then be integrated over $[t_p, t_p+T]$ assuming constant values $\xi_j^i(t) \equiv \hat{\xi}_j^i(t_p), i \in I_j$ and $t \in [t_p, t_p+T]$. The resulting predicted state trajectory is denoted by

$$X_j^p\big(\cdot, \hat{x}_j(t_p), \{\hat{\xi}_j^i(t_p)\}_{i\in I_j}, \mathbf{u_j}\big) \tag{15}$$

This corresponds to a predicted evolution of the indicator $\eta_j = V_j(x_j)$ that is given by:

$$\eta_j^p\big(\cdot, \hat{x}_j(t_p), \{\hat{\xi}_j^i(t_p)\}_{i\in I_j}, \mathbf{u_j}\big) = V_j(X_j^p(\cdot)) \tag{16}$$

Moreover, S_j compute predicted evolution $\hat{\eta}_i^j(\cdot)$ of all the indicators η_i, $i \in J_j$ by integrating the on-line identified models (11)-(12), namely:

$$\hat{\eta}_i^j\Big(\cdot, \bar{\eta}_i^j(t_p), h_{(i,j)}(X_j^p(\cdot))\Big) \quad ; \quad i \in J_j \tag{17}$$

In the remainder of this paper and in order to simplify the expressions, the whole data available at instant t at system S_j (either by communication from other systems or by its own observers) is denoted as follows:

$$D_j(t) = \Big(\hat{x}_j(t), \{\bar{\eta}_i^j(t_p)\}_{i\in J_j}, \{\hat{\xi}_j^i(t_p)\}_{i\in I_j}\Big) \tag{18}$$

Using the notation (18), the expressions (15), (16) and (17) can be rewritten in the following more condensed forms:

$$X_j^p\big(\cdot, D_j(t_p), \mathbf{u_j}\big) \;;\; \eta_j^p\big(\cdot, D_j(t_p), \mathbf{u_j}\big) \;;\; \hat{\eta}_i^j\big(\cdot, D_j(t_p), \mathbf{u_j}\big)$$

When parameterized control profile $\mathbf{u_j}(\cdot) = \mathcal{U}_{pwc}(\cdot, p)$ is used [see (14)], the above notation are slightly modified in order to express the fact that the decision variable p is used to completely define the control sequence:

$$X_j^p\big(\cdot, D_j(t_p), p\big) \;;\; \eta_j^p\big(\cdot, D_j(t_p), p\big) \;;\; \hat{\eta}_i^j\big(\cdot, D_j(t_p), p\big)$$

In the next sections, the NMPC scheme is defined by describing the cost function $J(D_j, \mathbf{u}_j)$ to be considered at system S_j given the available data D_j.

4.2 The rules governing the definition of the cost function

Recall that the variables η_i are defined such that the nominal value $\eta_i = 0$ reflects nominal and *desired* behavior of system S_i while values of η_i that are greater than 1 indicate that the system S_i is in a risky state.

The definition of the cost function is based on the following rules:

(1) The cost function used by system S_j takes the following form:

$$J_j(D_j, p) = J_j^{(0)}(D_j, p) + \sum_{i\in J_j} J_j^{(i)}(D_j, p) \tag{19}$$

where the first term, namely $J_j^{(0)}$ is related to the performance and stability of the system S_j itself while

[1] Note that by definition of the sets of indices I_i and J_j, one has $j \in I_i \Leftrightarrow i \in J_j$

[2] by the very definition of the η_i's that are linked to the control design positive function for systems S_i, $i \in J_j$ [see (10)]

the terms $J_j^{(i)}$ reflects the concerns that are related to the neighboring systems S_i that are influenced by S_j.

(2) If for some value of the control parameter p, the prediction of a neighbor indicator $\hat{\eta}_i^j$ for some $i \in J_j$ is under some low threshold $\eta^{(ok)} < 1$ over the whole prediction horizon, this system is considered (by S_j) in a safe situation that corresponds to a high level of performance. Therefore, for this value of p (and hence for the corresponding candidate control profile $\mathcal{U}_{pwc}(\cdot, p)$), the contribution of system S_i to the cost function J_j is equal to 0.

This rule can be written in a formal way as follows:

$$\left\{ \max_{t \in [t, t+T]} \left[\hat{\eta}_i^j(t, D_j(t), p) \right] \leq \eta^{[ok]} \right\} \quad \Rightarrow$$
$$\Rightarrow J_j^{(i)}(D_j(t), p) = 0 \tag{20}$$

(3) If at the decision instant t, the system S_j sees its own indicator η_j greater than some high value $\eta^{(danger)} \leq\approx 1$ then the NMPC solver first concentrates on system S_j safety by computing the minimum value of the following cost function:

$$J_j^{(danger)}(D_j, p) := \min_p \left[\max_{t \in [t, t+T]} \eta_j^p(D_j(t), p) \right]$$

if the optimal value, say $J_{j,opt}^{(danger)}(D_j)$, of the above optimization problem is greater than 1, then the contribution of the system S_j related term must vanish in the overall cost function S_j is using to compute its own control. This corresponds to a kind of sacrifice operated by S_j by shading itself in order to prevent useless effort and potentially protect the overall network from collapse. This rule can then be written as follows:

$$\left\{ J_{j,opt}^{(danger)}(D_j(t)) > 1 \right\} \Rightarrow J_j^{(0)}(D_j(t), p) = 0 \tag{21}$$

(4) For all neighboring systems S_i, $i \in J_j$ that *may need help*, that is for which $\hat{\eta}_i^j > \eta^{(ok)}$ [see rule (2) above], the contributions to the cost function are weighted by their relative current performance index $\hat{\eta}_i(t)$ that is known by S_j thanks to the communication rule 1. This can be written as follows:

$$J_j^{(i)}(D_j(t), p) :=$$
$$\rho_i(D_j(t)) \cdot \sum_{k=1}^{N} \hat{\eta}_i^j(t + k\tau_s, D_j(t), p) \tag{22}$$

where

$$\rho_i(D_j(t)) :=$$
$$\begin{cases} \dfrac{max\{0, \eta_i(t) - \eta^{(ok)}\}}{\eta^{(ok)} + \sum_{\sigma \in J_j} max\{0, \eta_\sigma(t) - \eta^{(ok)}\}} & \text{if } \hat{\eta}_i(t) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Note that the conditional definition of ρ_i incorporates the condition (20) by removing the contribution of system S_i as long as its predicted indicator is lower than some low threshold $\eta^{(ok)}$ during the prediction horizon. On the other hand, definition (23) enables systems S_i that are impossible to recover (for which $\hat{\eta}_i > 1$) to be removed from the cost function. Moreover, the definition (23) leads to the straightforward property:

$$\sum_{i \in J_j} \rho_i(D_j(t)) < 1$$

that enables the overall "cooperation" level of S_j to be controlled through the weighting factor c used in the definition of the cost function $J_j^{(0)}$ that can be taken equal to:

$$J_j^{(0)}(D_j(t), p) = c(D_j(t)) \cdot \sum_{k=1}^{N} \eta_j^p(t + k\tau_s, D_j(t), p) \tag{24}$$

where

$$c(D_j(t)) = \begin{cases} 0 & \text{if } J_{j,opt}^{(danger)}(D_j(t)) > 1 \\ c_0 & \text{otherwise} \end{cases} \tag{25}$$

Therefore, if $c_0 \gg 1$ is used, then the controller used in S_j has only a limited cooperative behavior with the neighboring systems S_i, $i \in J_j$. For lower values of $c_0 \approx 1$, strong cooperation is obtained. Note also that the conditional definition of $c(D_j(t))$ enables the *self-shading* rule (21) to be implemented when the indicator $\hat{\eta}_j$ is above the safety value 1.

4.3 Definition of the NMPC optimization problem

From the above discussion, it comes that the optimization problem used in the definition of the NMPC scheme at the system S_j level may be defined as follows:

(1) If $J_{j,opt}^{(danger)}(D_j(t)) < 1$ then, the optimization problem is given by:

$$\mathcal{P}_1 : \min_p \left[c_0 \cdot \sum_{k=1}^{N} \eta_j^p(t + k\tau_s, D_j(t), p) + \right.$$
$$\left. + \sum_{i \in J_j} \rho_i(D_j(t)) \cdot \sum_{k=1}^{N} \hat{\eta}_i^j(t + k\tau_s, D_j(t), p) \right]$$
$$\text{under } J_j^{(danger)}(D_j(t), p) \leq 1 \tag{26}$$

The condition $J_{j,opt}^{(danger)}(D_j(t)) < 1$ that underlines this formulation simply means that the above constrained optimization problem is feasible.

(2) Otherwise, the term related to S_j is removed and the following optimization problem is used:

$$\mathcal{P}_2 : \min_p \left[\sum_{i \in J_j} \rho_i(D_j(t)) \cdot \sum_{k=1}^{N} \hat{\eta}_i^j(t + k\tau_s, D_j(t), p) \right] \tag{27}$$

Regardless the problem being active, the solution of the corresponding optimization problem, say $p^{opt}(D_j(t))$, defines the optimal control sequence $\mathcal{U}_{pwc}\Big(\cdot, \hat{p}^{opt}(D_j(t)\Big)$ that is defined on the control horizon $[t, t+T]$. The first control of the optimal sequence is then applied leading to the feedback law:

$$\mathbf{u_j}(t+\sigma) = u^0\big(p_{opt}(D_j(t))\big) \quad \sigma \in [0, \tau_s] \qquad (28)$$

5. CONCLUSION AND FUTURE WORK

In this paper, a general scheme is proposed for the distributed partially cooperative control of a network of interconnected systems. The scheme is based on a simple and cheap communication rule that enables each subsystem to construct a prediction model for the evolution of key variables indicating each, the distance to instability of a neighboring system. This prediction is then used in a model predictive scheme that enables each system to balance its cooperation with its neighbor according to its own state and the situation of these neighbor as summarized by their estimated key quantities.

The basic assumption in the proposed scheme is the existence of nominal controllers at each subsystem level. This assumption is compatible and is even a preliminary working basis in many practical situation where the network related control has to deal with existing controllers.

A particular feature in the proposed framework is the possibility for a subsystem to shade-itself in order to make possible the overall network safety. This is again based on the prediction of the best possible evolution of the subsystem under the available amount of knowledge.

Future work concerns the instantiation of the proposed general framework to particular meaningful situations. Among them, the stabilization and more generally, the control of power transport networks appears as the first choice.

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