Constrained Minimum-time-oriented Stabilization of Extended Chained Form Systems

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***Abstract*— In this paper, a minimum-time oriented state feedback control is proposed for the stabilization of second-order chained form system with saturation constraints on the control inputs. The feedback law is based on receding horizon strategy that provides global stabilization of the system for any final desired state. Simulations are given to show the effectiveness of the proposed approach.**

I. INTRODUCTION

IN the past few years, the control of nonholonomic systems has attracted considerable amount of interest within the nonlinear control community. Many nonholonomic mechanical systems can be described by kinematic or dynamic models in chained form or a feedback equivalent to chained form. For instance mobile robots, under-actuated satellites, the knife-edge and car towing several trailers. Also the control of nonholonomic systems is extremely challenging and difficult for the reason that these systems do not satisfy the Brockett conditions [1]. These conditions are necessary conditions for the existence of time invariant C^1 static stabilizing feedbacks. Extensive results were obtained to first-order nonholonomic systems that undergo nonintegrable kinematic constraints [2] and can be classified under time varying asymptotic (smooth and non-smooth) [3], [4], [5], [6], [7], [8], [9], discontinuous [10], [11], [12] and hybrid feedbacks [13].

All of the above mentioned approaches are based on the Lyapunov theory. Very few publications deal with the stabilization of nonholonomic systems in an optimal way. In [14], a robust stabilization for the particular case of the planar vertical take-off and landing aircraft, close to the class of systems considered here, formulated as an optimization problem is proposed. However, the approach requires a partial linearization of the system and can be explicitly obtained only for very particular control parameters. The only predictive approach proposed seems to be [15] where a constrained minimum-time-oriented feedback control for the stabilization of nonholonomic systems in chained form is proposed and by which the present work is inspired. One of the interesting aspects of the approach is that it attaches a lot of importance to the real-time implementability of the scheme. This concern of applicability is also important in the present approach.

In this paper, we focus on a second-order nonholonomic system with saturation constraints on the control inputs as follows

$$\left\{\begin{array}{rcl} \ddot{x}_1 & = & u \\ \ddot{x}_2 & = & v \\ \ddot{x}_3 & = & x_2 u \end{array}\right. \tag{1}$$

$$-u^{max} \le u \le u^{max} \tag{2}$$

$$-v^{max} \le v \le v^{max} \tag{3}$$

and we define:

$$w := \left(\begin{array}{c} u \\ v \end{array} \right) \tag{4}$$

All the results proposed in the present paper can easily be generalized to higher derivative order and higher dimension systems. This system known as the extended chained form satisfies non-integrable relations involving not only generalized coordinates and velocities but also generalized accelerations. It differs from the chained form initially proposed in [16] by the way it contains a drift component. Typical examples of this system include unicycle-type vehicles, car-like vehicles and planar underactuated manipulators. Also V/STOL aircraft without gravity [17] can be transformed into a system that is equal to the second-order chained form using coordinate and feedback transformation [18].

This paper is organized as follows. In Section II, we present the open-loop optimization problem used to define the receding-horizon control with the formulation of the control input. In Section III the receding-horizon feedback is defined and a stability result is established. The paper ends with the simulation results presented in Section IV and the conclusion in Section V.

Notation: In this paper, $\delta > 0$ denotes some fixed sampling period. For any time-dependent signal $w(.)$, $w(k)$ or x_k simply denotes $w(k\delta)$. The classical notation $X(t;0;x_0;w)$ is used to denote the solution at instant t of system (1) starting from the initial state x_0 at initial time $t = 0$ under the control $w(.)$. Finally $W_{ad} := [-u^{max}, u^{max}] \times [-v^{max}, v^{max}]$ denotes the set of admissible controls.

II. OPEN-LOOP CONTROL PROBLEM

The feedback law proposed in this paper is based on the receding horizon principle. Recall that such a feedback is obtained by solving at each sampling instant $(k\delta)$ an open-loop optimization problem in which the current state $x(k)$ plays the role of initial state. Then the first part of the optimal control sequence is applied. At the next sampling instant $k+1$, a new open-loop optimization problem is solved with $x(k+1)$ as initial state, the first part of the resulting optimal sequence is applied and so on. The parametrization of the

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open-loop control profile over the future prediction horizon is defined by a strictly increasing sequence of n future instants, namely

$$t(\delta) = \begin{pmatrix} 0 & \delta & \dots & n\delta \end{pmatrix} \quad (5)$$

and the sequence of corresponding controls

$$W = \begin{pmatrix} U \\ V \end{pmatrix} = \left(\begin{pmatrix} u_0 \\ v_0 \end{pmatrix}, \begin{pmatrix} u_1 \\ v_1 \end{pmatrix}, \dots, \begin{pmatrix} u_{n-1} \\ v_{n-1} \end{pmatrix} \right) \quad (6)$$

The open-loop control profile therefore defines clearly a piecewise constant control over $[0, n\delta[$. The extended chained form system (1) can be written under the form of two single input subsystems. The first subsystem Σ_1 is LTI (linear time invariant) and takes the form of a double integrator chain with ζ as state vector and u as control input. The second subsystem Σ_2 is LTV (linear time-varying) if u is taken as a time function. z and v are respectively its state vector and control input.

$$\Sigma_1 : \dot{\zeta} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \zeta + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$\Sigma_2 : \dot{z} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v \quad (7)$$

where

$$\zeta = \begin{pmatrix} x_1 \\ \dot{x}_1 \end{pmatrix}, \; z = \begin{pmatrix} x_3 \\ \dot{x}_3 \\ x_2 \\ \dot{x}_2 \end{pmatrix}, \; x = \begin{pmatrix} \zeta \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ \dot{x}_1 \\ x_3 \\ \dot{x}_3 \\ x_2 \\ \dot{x}_2 \end{pmatrix} \quad (8)$$

A. The Open-loop Control Input Formulation

In this section, a clear expression of the parametrization (5-6) is presented. u_i and v_i will represent the control inputs to be applied to the system at the instant $i\delta$ to steer the system's state from x_i to x_{i+1}. The final goal of these control inputs is to steer system (1) from its initial state x_0 to the final desired state x_d.

1) Formulation of u: The proposed control aims to steer the state ζ of the first subsystem $\sum_1$ of (7) from an initial state to desired one after n sampling periods and is given by:

$$U = [u_0 \; u_1 \; \dots \; u_{n-1}]^T$$

such that

$$u_i = \alpha e^{-i\delta} + \beta e^{-2i\delta} \quad (9)$$

where α and β are functions of the initial state ζ_0 and the final desired states ζ_n. They can be determined by discretizing the first subsystem $\sum_1$ of (7) by taking δ as a sampling period to get:

$$\zeta_k = A_1 \zeta_{k-1} + B_1 u_{k-1} \quad (10)$$

with A_1 and B_1 are the discretization of Σ_1 and take the following form:

$$A_1 = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}; \quad B_1 = \begin{bmatrix} \frac{\delta^2}{2} \\ \delta \end{bmatrix} \quad (11)$$

The final desired state of Σ_1 can be reached starting from ζ_0 and by applying the corresponding control sequences of U to get:

$$\zeta_n = A_1^n \zeta_0 + \sum_{i=0}^{n-1} (A_1)^{n-1-i} B_1 u_i \quad (12)$$

Using u_i of equation (9), ζ_n can be written:

$$\zeta_n = A_1^n \zeta_0 + \Gamma \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (13)$$

with

$$\Gamma = [\sum_{i=0}^{n-1} (A_1)^{n-1-i} B_1 e^{-i\delta} \sum_{i=0}^{n-1} (A_1)^{n-1-i} B_1 e^{-2i\delta}] \quad (14)$$

Γ can be also written under the form of product of two matrices as follows, both being full rank.

$$[B_1 \; A_1 B_1 \; \dots \; A_1^{n-1} B_1] \begin{bmatrix} e^{-(n-1)\delta} & e^{-2(n-1)\delta} \\ \vdots & \vdots \\ e^{-0\delta} & e^{-0\delta} \end{bmatrix} \quad (15)$$

Hence Σ_1 is also full rank and thus invertible. So the parameters α and β are obtained using

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \Gamma^{-1} [\zeta_n - (A_1)^n \zeta_0] \quad (16)$$

The application of u_{1_i} (9) using the values of α and β obtained assures the steerage of subsystem Σ_1 from the initial state ζ_0 to the desired final state ζ_n in n sampling periods.

2) Formulation of v: In order to find v_i the control input of the second subsystem Σ_2, we discretize Σ_2 using the same sampling period δ. The discrete states of Σ_2 can be represented by:

$$\begin{cases} z_1 &= A_{2_0} z_0 + B_{2_0} v_0 \\ z_2 &= A_{2_1} z_1 + B_{2_1} v_1 \\ \vdots & \\ z_n &= A_{2_{n-1}} z_{n-1} + B_{2_{n-1}} v_{n-1} \end{cases} \quad (17)$$

where A_{2_i} and B_{2_i} are obtained discretizing Σ_2 after having replaced u_i by its value at time $i\delta$ and taking δ as sampling period. The final desired state z_n can be expressed by:

$$z_n = \Phi_n z_0 + \Psi_n V \quad (18)$$

where Φ_n, Ψ_n, and V are given by

$$\Phi_n = \prod_{j=1}^{n} A_{2_{j+1}} \quad (19)$$

$$\Psi_n = \big(A_{2_{n-1}} A_{2_{n-2}} \cdots A_{2_1} B_{2_0}, \; A_{2_{n-1}} A_{2_{n-2}} \cdots A_{2_2} B_{2_1}, \dots \; A_{2_{n-1}} B_{2_{n-2}}, \; B_{2_{n-1}} \big) \quad (20)$$

$$V = (v^0 \; v^1 \; \dots \; v^{n-1})^T \quad (21)$$

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For all $u \neq 0$, $\sum_2$ is controllable insuring that Ψ_n has a rank of four. Hence, the control sequence V is defined by

$$V = \Psi_n^+ (z_n - \Phi_n z_0) \tag{22}$$

where Ψ_n^+ denotes the Moore-Penrose Pseudo inverse of Ψ_n, is admissible in the sense that it steers Σ_2 from the initial state z_0 to z_n in n sampling periods. The obtained control input denoted by W steers system (1) from an initial state x_0 to any final desired state x_d without taking the saturation constraints (2-3) into consideration. The aim of the next section is to propose an algorithm that iteratively converges to control sequences that fulfill the constraints.

3) Algorithm $A(x^0)$: The objective of using this algorithm is to find u_i and v_i belonging to W_{ad} thus respecting the saturation constraints. The inputs of this algorithm are the initial state x_0 and the final desired state $x_d = (\zeta_n, z_n)$. This algorithm is based on the following steps.

1) Take an initial future prediction horizon n.
2) Calculate Γ, Φ_n and Ψ_n using (14), (19) and (20) respectively.
3) Compute U and V using (16) and (22).
4) Test if the elements of U and V belong to the admissible region.
5) If the test fails, increase the prediction horizon by one period δ: $n = n + 1$.
6) The algorithm is executed from step (2) until u_i and v_i, that respect the saturation constraints, are found.

Let $\hat{W}$ denotes the control vector obtained after the execution of the above algorithm

$$\hat{W} = \begin{pmatrix} \hat{U} \\ \hat{V} \end{pmatrix} = \left(\begin{pmatrix} \hat{u}_0 \\ \hat{v}_0 \end{pmatrix}, \begin{pmatrix} \hat{u}_1 \\ \hat{v}_1 \end{pmatrix}, \dots, \begin{pmatrix} \hat{u}_{n-1} \\ \hat{v}_{n-1} \end{pmatrix} \right) \tag{23}$$

it contains the admissible control sequences to be applied during the time

$$\hat{t}_{\hat{n}}(x_0) = (0 \quad \delta \quad \dots \quad \hat{n}\delta) \tag{24}$$

where $\hat{n}$ denotes the length of the future prediction horizon. The control inputs derived from the above algorithm assures the steerage of the extended chained form system from the initial state x_0 to the final desired state x_d in n sampling periods with fulfilling the saturation constraints.

In the following, we will discuss the singular situation takes place when $u = 0$ that leads to lose the controllability of Σ_2 and the whole system. This singular case arises when α and β of u (16) equal zero. In other words, when $\zeta_n = (A_1)^n \zeta_0$.

4) Singular case if $u = 0$: In order to avoid the singular situation when $u = 0$, an additional step is first applied before executing the above algorithm. This step also helps to increase the convergence speed of the system form the initial state x_0 to the final desired state x_d. So a constant control parameterized by an integer $q \in \{0, \dots, q_{max}\}$ and a discrete variable $\varepsilon \in \{-1, 1\}$ is first applied.

$$w_{q,\varepsilon}(\tau) = \begin{pmatrix} \varepsilon u_1^{max} \\ 0 \end{pmatrix} \qquad \forall \tau \in [0, q\delta] \tag{25}$$

The application of this control during q sampling periods steers the system's state to $x_f(x_0, q, \varepsilon) = X(q\delta; 0; x_0; w_{q,\varepsilon}(.))$ which itself is steered to the final desired state x_d by the application of the algorithm $A(x_f)$.

An admissible sub-optimal open-loop control is obtained by solving the following optimization problem

$$(\hat{q}(x_0), \hat{\varepsilon}(x_0)) = \text{Arg} \min_{(q,\varepsilon) \in \mathcal{A}(x_0)} q\delta + \hat{t}_{\hat{n}}(x_f(x_0, q, \varepsilon)) \tag{26}$$

where

$$\begin{aligned} \mathcal{A}(x_0) = \{(q, \varepsilon) \in \{0, \dots, q_{max}\} \times \{-1, 1\} \quad | \\ \hat{t}_{\hat{n}}(x_f(x_0, q, \varepsilon)) < \hat{t}_{\hat{n}}(x_0) - q.\delta\} \end{aligned} \tag{27}$$

The overall retained solution denoted by $(t^{opt}(x_0), u^{opt}(x_0))$ is then given by

$$t^{opt}(x_0) = (0, \delta, \dots, \delta\hat{q}(x_0), \hat{t}_{\hat{n}}(x_f)) \tag{28}$$

$$w^{opt}(x_0) = (w_0^{opt}, \dots, w_{\hat{q}(x_0)}^{opt}, \hat{W}(x_f)) \tag{29}$$

with, for $j = 1, \dots, \hat{q}(x_0)$:

$$w_j^{opt} = \begin{pmatrix} \hat{\varepsilon}(x_0) u^{max} \\ 0 \end{pmatrix}$$

and

$$\hat{t}_{\hat{n}}(x_f) = ((\hat{q}(x_0) + 1)\delta, \dots, (\hat{q}(x_0) + \hat{n}(x_0))\delta) \tag{30}$$

$\hat{W}(x_f)$ is the control obtained from the execution of algorithm $A(x_f)$. In the following section, the receding-horizon state feedback control is defined and a stability result of the closed-loop system is given.

III. FEEDBACK DEFINITION AND STABILITY RESULTS

Using the previous notations and algorithm we can state the following theorem

Theorem 1: The following discrete-time state feedback law defined for all $\sigma \in [0, \delta[$ and having the following expression

$$w(k\delta + \sigma) = W(0, t^{opt}(x_k), w_0^{opt}(x_k)) \tag{31}$$

assures the global stabilization of system(1) and steers it from any given initial state x_0 to final desired state x_d.

To prove theorem 1, we will begin in subsection III-A to establish a transitivity property that insures the Bellman invariance principle of the proposed scheme. In other words, we will check that, for all k, the optimal solution at time $k\delta$ remains, shifted by one sample period, in the set of control profile where the optimal solution for time $(k + 1)\delta$ is searched. This property enables us to conclude to stability results in subsection III-B. Let $x_{cl}(.)$ denotes the closed-loop system trajectories under (31).

A. The Transitivity Property

This property is crucial for the proof of the stability of the proposed control. Indeed, the control law is here obtained by computing u and then v. Hence, if u is not at time $(k+1)\delta$ the same as at time $k\delta$ shifted by one period, this will totally change the optimization problem and the space where the solutions are searched for v. The stability will then be hard to prove. Suppose we want to steer a state from ζ_k of Σ_1 of (7) at instant $k\delta$ to ζ_d, the corresponding control input will take the following form

$$u_j^k = \alpha_k e^{-j\delta} + \beta_k e^{-2j\delta} \tag{32}$$

The objective is to find a relation between (α_k, β_k) and $(\alpha_{k+1}, \beta_{k+1})$ which corresponds to the application of u_j^{k+1} on ζ_{k+1} to drive the state trajectory to the desired state ζ_d. We have

$$\Gamma_k \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} = [\zeta_d - (A_1)^{n-k}\zeta_k] \tag{33}$$

with

$$\Gamma_k = [\sum_{i=0}^{n-k-1} (A_1)^{n-k-1-i} B_1 e^{-i\delta} \quad \sum_{i=0}^{n-k-1} (A_1)^{n-k-1-i} B_1 e^{-2i\delta}] \tag{34}$$

also we have

$$\Gamma_{k+1} \begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} = [\zeta_d - (A_1)^{n-k}\zeta_{k+1}] \tag{35}$$

with

$$\Gamma_{k+1} = [\sum_{i=0}^{n-k-2} (A_1)^{n-k-2-i} B_1 e^{-i\delta} \quad \sum_{i=0}^{n-k-2} (A_1)^{n-k-2-i} B_1 e^{-2i\delta}] \tag{36}$$

Knowing that

$$\zeta_{k+1} = A_1 \zeta_k + B_1 u_k^0 \tag{37}$$

and using (34,36) we will finally get the following relation

$$\begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_k e^{-\delta} \\ \beta_k e^{-2\delta} \end{bmatrix} \tag{38}$$

This relation implies with (9) that the application of u^k which steers the state trajectory from x_k to x_d in a time $(n-k)\delta$ is similar to the application of u^{k+1} where the state trajectory is steered from x_{k+1} to x_d in $(n-k-1)\delta$.

B. Stability Results

Proposition 1: Let $\delta > 0$ be fixed. There is a scalar function $V : \mathbb{R}^n \to \mathbb{R}^+$ with the following properties

1) $(V(x) = 0) \Leftrightarrow (x = x_d)$
2) V is radially unbounded.
3) For all $k \in \mathbb{N}$,

$$V(x_{cl}(k+1)) - V(x_{cl}(k)) \le -\delta \quad \text{whenever } V(x_{cl}(k)) > \delta \tag{39}$$

Proof: It will be shown that V defined by

$$V(x) = \min_{i \in \{1,\dots,n\}} \{t_i^{opt}(x) \mid X(t_i^{opt}(x); 0; x; W(., t^{opt}(x_0), w^{opt}(x_0))) = 0\} \tag{40}$$

satisfies the items of proposition 1. Note that because of the steering property of $(t^{opt}(x), w^{opt}(x))$, we know that $V(x)$ is well defined and such that $V(x) \le t_n^{opt}(x)$.
Item 1) results directly from the fact that under bounded control, the system state cannot be steered from $x_0 \neq x_d$ to x_d infinitely fast.
Item 2) results from the fact that under bounded control, the time necessary to steer the state from x_0 to x_d tends to infinity when $\|x_0\|$ tends to infinity.
To prove item 3) we use (26) as a cost function namely

$$J(q, \varepsilon, x_0) = q\delta + \hat{t}_n(x_f(x_0, q, \varepsilon)) \tag{41}$$

and let us use $(\hat{q}_k, \hat{\varepsilon}_k)$ to denote $(\hat{q}(x_{cl}(k)), \hat{\varepsilon}(x_{cl}(k)))$. Two cases are to be considered:
Case 1: $\hat{q}_k > 0$. In this case, define $x^+(k) := X(\hat{q}_k\delta; 0; x_{cl}(k); u_{\hat{q}_k, \hat{\varepsilon}_k}(.))$. By definition, $V(x_{cl}(k))$ is given by

$$V(x_{cl}(k)) = \hat{q}_k\delta + \hat{t}_{i_0}(x^+(k)) \quad \text{for some } i_0 \le n \tag{42}$$

Using the transitivity property found in section (III-A), we can get

$$X(\tilde{q}_{k+1}\delta; 0; x_{cl}(k+1); w_{\tilde{q}_{k+1}, \tilde{\varepsilon}_{k+1}}(.)) = x^+(k)$$

with the sub-optimal solution

$$(\tilde{q}_{k+1}, \tilde{\varepsilon}_{k+1}) = (\hat{q}_k - 1, \hat{\epsilon}_k)$$

Therefore, by (42)

$$\begin{aligned} V(x_{cl}(k+1)) &\le J(\tilde{q}_{k+1}, \tilde{\varepsilon}_{k+1}, x_{cl}(k+1)) \\ &\le (\hat{q}_k - 1)\delta + \hat{t}_{i_0}(x^+(k))) \\ &\le V(x_{cl}(k)) - \delta \end{aligned}$$

which is nothing but (39).
Case 2: $\hat{q}_k = 0$. In this case, the next state on the closed loop trajectory is clearly given by $x_{cl}(k+1) = X\Big(\delta; 0; x_{cl}(k); W(0, t^{opt}(x_k), w_0^{opt}(x_k))\Big)$. Now, choosing the sub-optimal solution $(\tilde{q}_{k+1}, \tilde{\varepsilon}_{k+1}) = (0, \hat{\epsilon}_k)$ implies that

$$J(\tilde{q}_{k+1}, \tilde{\varepsilon}_{k+1}, x_{cl}(k+1)) = \hat{t}_n(x_{cl}(k+1)) \tag{43}$$

but from (III-A) one has

$$\hat{t}(x_{cl}(k+1)) = \hat{t}(x_{cl}(k)) - \delta \tag{44}$$

This with (43) shows that whenever $V(x_{cl}(k)) > \delta$, the resulting steering time from $x_{cl}(k+1)$ to x_d is lower by at least δ than the steering time from $x_{cl}(k)$ to x_d. This clearly gives (39). ∎

IV. SIMULATION

In this section, 3 simulation studies are carried out to demonstrate the effectiveness of the proposed controller. In all the simulations, the sampling period is given by $\delta = 0.3\,s$, the initial state of the system $x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T$ and the future prediction horizon $n.\delta$ is initialized for $n = 6$. In the first and second simulation the input control constraint is $[u^{max}, v^{max}] = [12, 10]$ and final desired state is $x_d = [0, 0, 0, 0, 0, 0]^T$. The first simulation corresponds to the application of the control using the additional step of (II-A.4) with $q_{max} = 3$ while in the second simulation, the control is applied without using (II-A.4). We can observe that the application of control of (II-A.4) increases the convergence speed. In third simulation the input control constraints are $[u^{max}, v^{max}] = [2.5, 2.5]$ and the system will be steered to the desired state $x_d = [1, 0, 1, 0, 1, 0]^T$. This simulation shows that the proposed controller works for any desired state different from the origin (Fig. 7 and 8). Note also that the constraints have been strengthened, making the control law converge to a bang-bang behaviour well known in time optimal control.

V. CONCLUSION

In this paper, a state feedback control which assures the stabilization of the extended chained form system and that respects the saturation constraints on the control inputs is proposed. The advantage of the proposed controller is that it overcomes the singular situations and stabilizes the system in minimum time manner.

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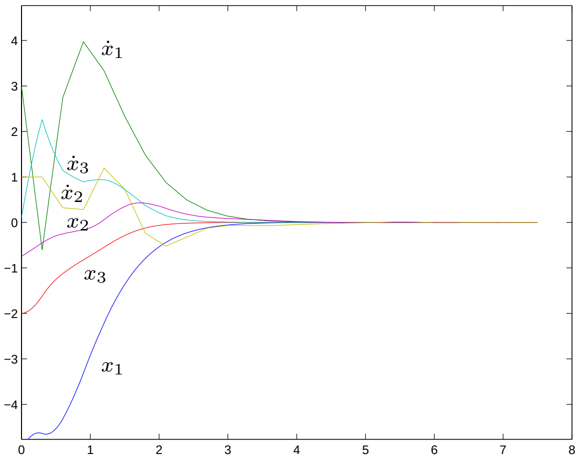

Fig. 1. The evolution of the state from initial state $x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T$ to the desired state $x_d = [0, 0, 0, 0, 0, 0]^T$ with the application of the additional step of (II-A.4)

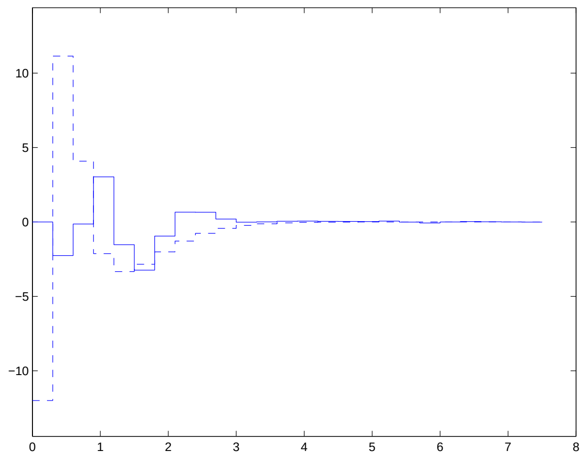

Fig. 2. The control inputs u (dashed line) and v (continuous line) with $[u^{max}, v^{max}] = [12, 10]$ obtained from the application of the algorithm $A(x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T)$ plus the additional step of (II-A.4)

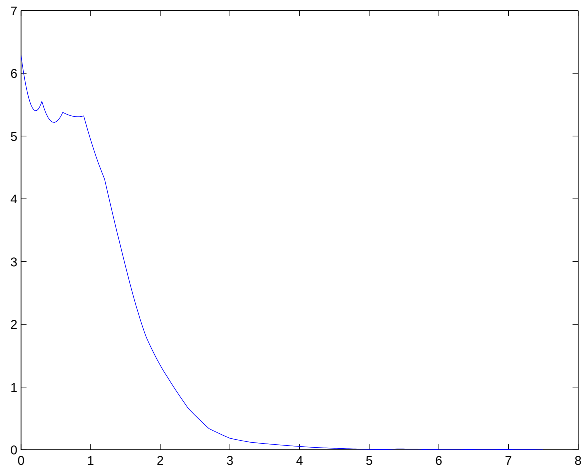

Fig. 3. The state norm under the control presented in Fig. (2)

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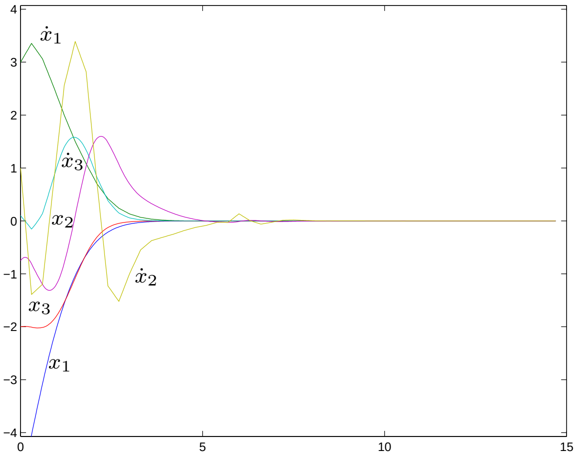

Fig. 4. The evolution of the state from initial state $x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T$ to the desired state $x_d = [0, 0, 0, 0, 0, 0]^T$ without applying the improvement presented in (II-A.4)

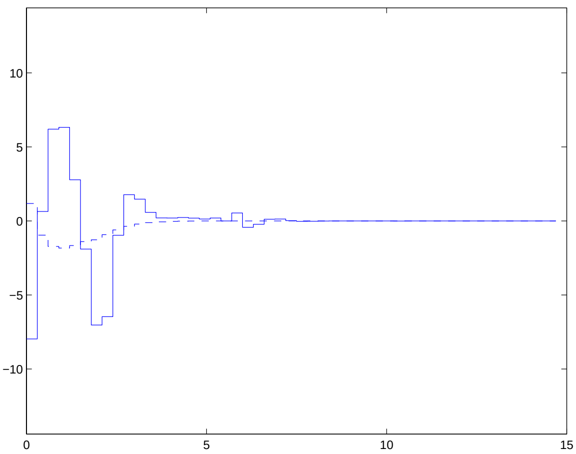

Fig. 5. The control inputs u (dashed line) and v (continuous line) with $[u^{max}, v^{max}] = [12, 10]$ obtained from the application of the algorithm $A(x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T)$

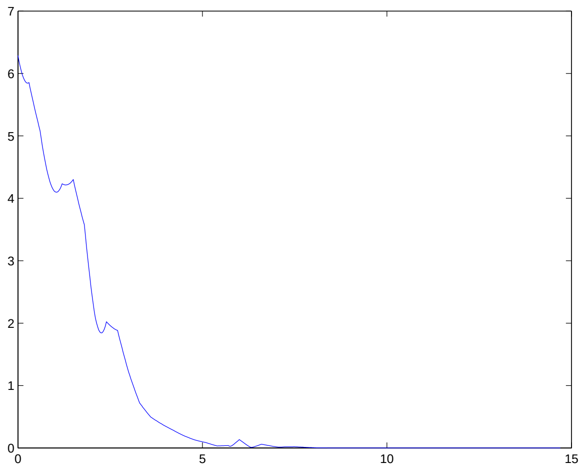

Fig. 6. The state norm under the control presented in Fig. (5)

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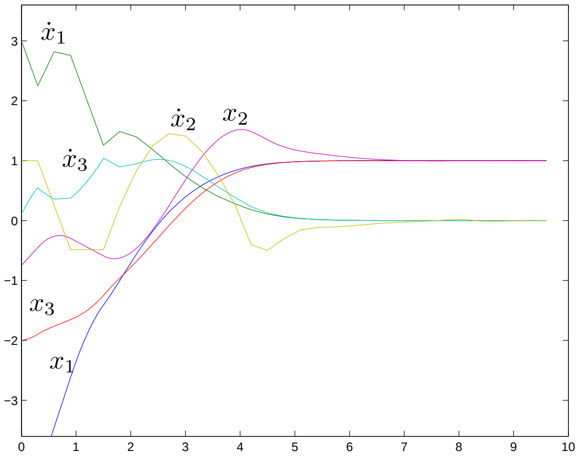

Fig. 7. The evolution of the state from initial state $x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T$ to the desired state $x_d = [1, 0, 1, 0, 1, 0]^T$ under the saturation constraints $[u^{max}, v^{max}] = [2.5, 2.5]$

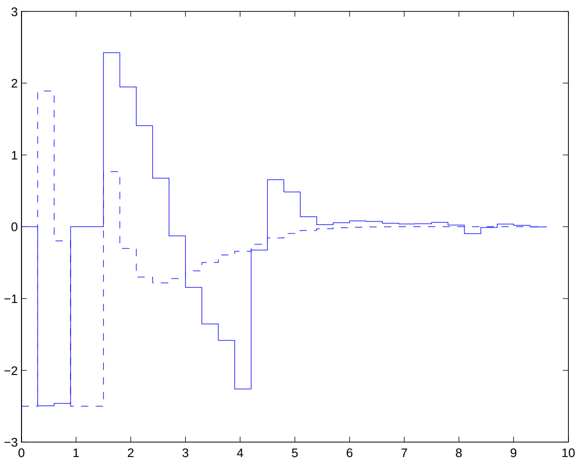

Fig. 8. The control inputs u (dashed line) and v (continuous line) with $[u^{max}, v^{max}] = [2.5, 2.5]$ that steers the system from initial state $x_0 = [-5, 3, -2, 0.1, -0.75, 1]^T$) to the desired state $x_d = [1, 0, 1, 0, 1, 0]^T$