NONLINEAR PREDICTIVE CONTROLLER FOR THE SIMPLIFIED ABB TEST POWER SYSTEM STABILIZATION PROBLEM*

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ABSTRACT

A nonlinear predictive controller is proposed to solve the simplified ABB test problem. Interesting feature of the proposed control law is its ability to be easily generalizable to the original problem. Many validating scenarios are proposed to illustrate the effectiveness of the proposed approach.

KEYWORDS: Voltage stabilization, Predictive Control, Nonlinear Hybrid Switched Systems.

1 INTRODUCTION

The ABB benchmark is a power system voltage stabilization problem that has been defined in the context of the CC-Euoprean project in order to illustrate control strategies on hybrid systems.

Preliminary studies showed high sensitivity of open-loop behaviors to control parametrization. This suggested the use of a more simplified model, at least in a first step, in order to gain a deeper insight into both the simplified and the original problem.

The work proposed in this paper treats the simplified test model problem that has been elaborated in [1] as an issue to the Ascona meeting in October 2002. It is shown that by appropriately parameterizing the control, it is possible to find open-loop control profiles that steer the simplified system (that has been destabilized after a change in the transmission network's reactance) to a new stable regime while being compatible with the tap changer discrete mechanism as well as the available computation time. The resulting steering open-loop control profiles are then used in a predictive control scheme resulting in a stabilizing feedback controller.

The paper is organized as follows. First, the equations of the simplified model are briefly recalled in section 2. Then the proposed algorithm is detailed in section 3. Finally, section 4 shows numerical simulations illustrating the effectiveness of the proposed algorithm in stabilizing the simplified model while handling the relatively non standard control specifications.

2 THE SIMPLIFIED MODEL

The simplified system considered in this paper is depicted on figure 2. According to [1, 2], focusing on tap changer dynamics (slow when compared to the generator dynamics), the generator bus can be modelled as an infinite bus. The load is modelled as an exponential recovery load, namely

$$\dot{x}_p = -\frac{x_p}{T_p} + P_0(1-v^2) \quad ; \quad \dot{x}_q = -\frac{x_q}{T_q} + Q_0(1-v^2)$$
$$P_d = (1-k)(x_p/T_p + P_0 v^2) \quad ; \quad Q_d = (1-k)(x_q/T_q + Q_0 v^2) \tag{1}$$

where P_d is the actual active load power, Q_d the actual reactive load power while T_d and T_q are the corresponding recovery time constants. The transmission network is simply modelled as a pure reactance X in series with an ideal transformer. Note that the parameter $(1-k)$ has been introduced

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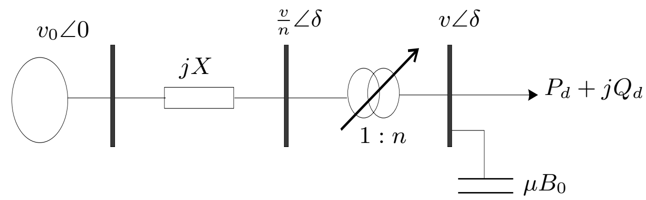

Figure 1: The simplified system

in (1) to model load shedding that is to be considered as a lost resort control action. Note then that k must satisfy the following saturation-type constraint

$$k \in \mathcal{K} := \{0\ ,\ 0.05,\ 0.1\ ,\ \dots k_{max}\} \tag{2}$$

Now, the active and the reactive powers supplied to the load are clearly given by [1]

$$P_d = -\frac{v_0 v}{nX}\sin\delta \quad ; \quad Q_d = \frac{v_0 v}{nX}\cos\delta - \frac{v^2}{n^2 X} + \mu B_0 v^2 \tag{3}$$

where the term $\mu B_0 v^2$ represents the reactive power in the capacitor bank (see figure 2). Note that B_0 is a constant while μ is a control action representing relative change in the capacitor bank resulting value. The control input μ is constrained to belong to the following discrete set[1]

$$B_0 = 0.2 \quad ; \quad \mu \in \mathcal{M} := \Big\{0.25\ ,\ 0.75\ ,\ 1\ ,\ 1.25\ ,\ 1.5\Big\} \tag{4}$$

this clearly leads to the following discrete set of admissible capacitors using the following notations $x := \begin{pmatrix} x_p \\ x_q \end{pmatrix}$, $y := v$ and $u := (n, \mu, k)^T$, the system model can be written as follows

$$\dot{x} = f(x, y) \quad ; \quad g(x, y, u) = 0 \tag{5}$$

It is worth noting that because of the tap changer dynamics, the transformer ratio n cannot be rigourously considered as a control variable. The true control variable must be v_r that is used in the tap changer definition (see figure 2). However, it is clear that if the following three conditions are respected

- The open-loop control input $n(.)$ is piece-wise constant with a sampling period T_d (where T_d is the time delay of the OLTC model)
- The piece-wise constant open-loop control input $n(.)$ is such that for all $j \in \mathbb{N}$, one has

  $$|n(j+1) - n(j)| = d_n \quad (= 0.02)$$

  where d_n is the step size modulus applied to n by the OLTC model when entering the "*action*" mode.
- The piece-wise constant open-loop control input $n(.)$ is such that for all $j \in \mathbb{N}$, one has

  $$(0.8 =) \quad n_{min} \le n(j) \le n_{max} \quad (= 1.2)$$

then it is possible to use n as a constrained control variable and to obtain v_r by inverting the dynamic of n. This is done by taking for all $t \in [jT_d, (j+1)T_d[$

$$v_r(t) = v(t) - u_f \times Sign(n(j+1) - n(j)) \tag{6}$$

Definition 1 *A sequence* $\Big(n(j)\Big)_{j\in\mathbb{N}}$ *that meets the above three requirements is said to be tap-changer compatible.*

[1] This choice is arbitrary and the proposed algorithm holds for any choice of such a discrete set. The set defined in (4) while realistic is only an illustrative choice

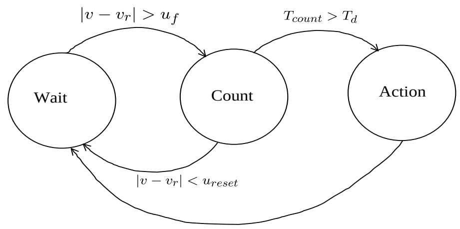

Figure 2: The OLTC control system

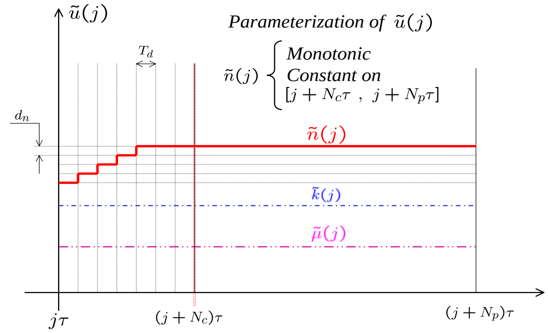

Figure 3: Parametrization of the open-loop profiles

3 A NONLINEAR PREDICTIVE CONTROLLER

Nonlinear predictive control [3] is now widely recognized to be a feedback strategy providing a relatively easy handling of both nonlinearities, constraints and optimality concerns. Recall that predictive control schemes amount to compute at each sampling time an optimal open-loop control sequence (in the sense of some given cost function), to apply the first part of the resulting optimal open-loop control sequence until the next sampling instant. At the next sampling instant, the whole problem is re-considered on a moving-horizon and the procedure is repeated indefinitely resulting in a state feedback law. For nonlinear systems and for long prediction horizons, this may lead to open-loop optimal control problems with a high dimension of the decision variable. This together with the non convex nature of the problem may render the on-line computations necessary to implement the resulting strategy unfeasible. That is the reason why a key feature in nonlinear predictive control design lies in the choice of control parametrization that leads to reasonably reduced complexity. The aim of the following subsection is to clearly define a reduced dimensional parametrization of open-loop controls that are used afterward in the predictive control implementation. Recall that a key feature in receding-horizon control is that the resulting closed-loop control is much more rich than the underlying open-loop parametrization. The consequence of this is that in many cases, apparently over-simplified open-loop parameterizations results in a sufficiently rich closed-loop control behavior.

3.1 The open loop control parametrization

Recall that the control is implemented in a sampling scheme with a sampling period $\tau = T_d$ and that based on this assumption, the control input in our problem is given at each sampling instant $j\tau$ ($\tau = T_d$) by

$$u(j) \quad := \quad \begin{pmatrix} n(j) \\ \mu(j) \\ k(j) \end{pmatrix} \in \Big\{ n(j-1) - d_n \ , \ n(j-1) \ , \ n(j-1) + d_n \Big\} \times \mathcal{M} \times \mathcal{K} \tag{7}$$

with the constraint $n_{min} \leq n(j) \leq n_{max}$. Recall that the sets $\mathcal{M}$ and $\mathcal{K}$ have been defined in (4) and (2) respectively.

Like any predictive control scheme, one must first define a prediction horizon. Given some sampling time, this prediction horizon may be defined as an integer number N_p of sampling periods (see Figure 2). To define the control parametrization, one has to clearly define at each sampling instant $j\tau$ the structure of the control $u(\cdot)$ over the time horizon $[j\tau, (j + N_p)\tau]$. Before, let us use the following notations to denote the open-loop control profiles at instant $j\tau$ over the prediction horizon $[j\tau, (j + N_p)\tau]$:

$$\tilde{u}(j) \quad := \quad \big(u(j\tau), \ldots, u\big((j + N_p - 1)\tau\big) \big) \quad ; \quad \tilde{n}(j) := \big(n(j\tau), \ldots, n\big((j + N_p - 1)\tau\big) \big)$$

$$\tilde{\mu}(j) \quad := \quad (\,\mu\big(j\tau\big),\dots,\mu\big((j+N_p-1)\tau\big)\,) \quad ; \quad \tilde{k}(j) := (\,k\big(j\tau\big),\dots,k\big((j+N_p-1)\tau\big)\,)$$

The control parametrization is defined by adopting the following choices

- ✓ The control profiles $\tilde{k}(j)$ and $\tilde{\mu}(j)$ are constant.
- ✓ The control profiles $\tilde{n}(j)$ is monotonic starting from $n(j-1)$
- ✓ The control profiles $\tilde{n}(j)$ is constant on $[(j+N_c)\tau,(j+N_p)\tau]$

The last two points define $2N_c+1$ possible choices for $\tilde{n}(j)$. N_c increasing profiles, N_c decreasing profiles and 1 constant profiles. As a result, the resulting parametrization leads to a decision variable of dimension $(2N_c+1)\cdot card(\mathcal{M})\cdot card(\mathcal{K})$. Note also that all candidate sequences $\tilde{n}(j)$ defined by the above rules are tap changer-compatible in the sense of definition 1.

3.2 Further definitions and notations

In order to properly present the optimal control computation, some further definitions and notations are needed.

Let $\tilde{\mathcal{U}}(n(j-1))$ be the set of admissible profiles at instant $j\tau$. The fact that this set depends on the past value of n results from (7). This will be shortly denoted by $\tilde{\mathcal{U}}_j$. Sometimes, the index j is omitted when no ambiguity follows.

We shall define an equivalence relation on $\tilde{\mathcal{U}}_j$ by

$$\Big\{\tilde{u}^{(1)}\sim\tilde{u}^{(2)}\Big\}\Leftrightarrow\Big\{(\tilde{\mu}^{(1)},\tilde{k}^{(1)})=(\tilde{\mu}^{(2)},\tilde{k}^{(2)})\Big\} \tag{8}$$

The need for such equivalence relation comes from the fact that while constraints are imposed on the use of capacitor changes (μ) or load shedding (k), no explicit constraint is considered to state that such tap changer compatible sequence $\tilde{n}^{(1)}$ is better or worst that some other one, still compatible sequence $\tilde{n}^{(2)}$. Denote by $\mathcal{U}_j^{eq}:=\Big\{\tilde{\mathcal{U}}_j^{(\mu,k)}\Big\}_{(\mu,k)\in\mathcal{M}\times\mathcal{K}}$ the set of equivalence classes that partitions $\tilde{\mathcal{U}}_j$ according to the equivalence relation (8), namely, an equivalent class (an element of $\mathcal{U}_j^{eq}$) is defined by the corresponding (μ,k) pair

$$\tilde{\mathcal{U}}_j^{(\mu,k)}:=\Big\{\tilde{u}=(\tilde{n},\tilde{\mu},\tilde{k})\in\tilde{\mathcal{U}}_j \quad | \quad \tilde{\mu}\equiv\mu \quad \text{and} \quad \tilde{k}\equiv k\Big\} \tag{9}$$

Let $\mathcal{O}:\mathcal{M}\times\mathcal{K}\rightarrow\Big\{1,\dots,card(\mathcal{M})\cdot card(\mathcal{K})\Big\}$ be a numbering of $\mathcal{U}_j^{eq}$ that reflects the priority in the choice of the control action. Namely

- $\Big\{k^{(1)}>k^{(2)}\Big\}\Rightarrow\Big\{\mathcal{O}(\mu^{(1)},k^{(1)})>\mathcal{O}(\mu^{(2)},k^{(2)})\Big\}$
- $\Big\{k^{(1)}=k^{(2)} \text{ and } \Big\{|\mu^{(1)}-\mu_0|>|\mu^{(2)}-\mu_0|\Big\}\Big\}\Rightarrow\Big\{\mathcal{O}(\mu^{(1)},k^{(1)})>\mathcal{O}(\mu^{(2)},k^{(2)})\Big\}$

where μ_0 is a reference value reflecting the nominal capacitors configuration ($\mu_0=1$ is used in the simulations hereafter). This order relation reflects the concern that changes in the capacitors have to be preferred rather than load shedding if the control n is unable to lonely recover the voltage collapse.

We shall denote by $X(t;x_0,\tilde{u})$ the solution of the system's equations starting from the initial conditions $(0,x_0)$ and under the control sequence $\tilde{u}$. Using this notation, the following two sets are defined (subscript "nc" is used for *no collapse*)

$$S_{nc}(x_0,j):=\Big\{(\mu,k)\in\mathcal{M}\times\mathcal{K} \quad | \quad \exists\tilde{u}\in\tilde{\mathcal{U}}_j^{(\mu,k)}:X(\cdot,x_0,\tilde{u}) \quad \text{is defined over } [0,N_p\tau]\Big\} \tag{10}$$

in other words, $S_{nc}(x_0, j)$ is the set of pairs (μ, k) for which there is a control sequence belonging to the class $\tilde{\mathcal{U}}_j^{(\mu,k)}$ such that starting from x_0, no voltage collapse occurs on the prediction time interval $[0, N_p\tau]$.

Finally, the following subset of $S_{nc}(x_0, j)$ needs to be defined

$$S_\varepsilon(x_0, j) := \left\{ (\mu, k) \in S_{nc}(x_0, j) \quad | \quad \exists \tilde{u} \in \tilde{\mathcal{U}}_j^{(\mu,k)} : \quad \forall t \in [0, N_p\tau] \quad y(t, x_0, \tilde{u}) \in [1-\varepsilon, 1+\varepsilon] \right\}$$

This is the subset of $S_{nc}(x_0, j)$ (if any) for which the resulting voltage meets the precision requirement over the whole prediction interval.

3.3 The feedback definition

Having at hand the notations of the preceding section, the proposed nonlinear predictive control may be properly defined. Let the pair $(\mu^*(x, j), k^*(x, j)) \in \mathcal{M} \times \mathcal{K}$ be given by

$$(\mu^*(x, j), k^*(x, j)) := \begin{cases} min_{(\mu,k)\in S_\varepsilon(x,j)} \mathcal{O}(\mu, k) & \text{if } S_\varepsilon(x, j) \neq \emptyset \\ min_{(\mu,k)\in S_{nc}(x,j)} \mathcal{O}(\mu, k) & \text{if } S_\varepsilon(x, j) = \emptyset \end{cases} \tag{11}$$

Note that since k_{max} is taken equal to 1, $S_{nc}(x, j)$ is never empty. Again, if $k > 0.15$ is needed to recover the voltage, then, either this limit is fictitious and higher values are anyway chosen, or this is a real limitation and in this case, the proposed feedback (or any other feedback ?) fails to recover the voltage since collapse occur before $N_p\tau$. Based on $(\mu^*(x, j), k^*(x, j))$, the "*optimal open-loop*" control profile at instant $j\tau$ is obtained by solving the following optimization problem

$$\hat{u}(x(j), j) := \text{Arg} \min_{\tilde{u} \in \tilde{\mathcal{U}}_j^{(\mu^*(x(j),j), k^*(x(j)))}} J(\tilde{u}, x(j), j) \tag{12}$$

$$=: \quad (\, \hat{u}_0(x(j), n(j-1)) \quad \ldots \quad \hat{u}_{N_p-1}(x(j), n(j-1))\,) \in \tilde{\mathcal{U}}_j \tag{13}$$

where $J(\tilde{u}, x(j), j) := \int_0^{N_p\tau} |y(\tau, x(j), \tilde{u}) - 1|^2 d\tau$ yielding the following state feedback law

$$u(t) = \hat{u}_0(x(j), n(j-1)) \quad ; \quad \forall t \in [j\tau, (j+1)\tau) \tag{14}$$

4 SOME VALIDATING SIMULATIONS

In this section, some simulations are proposed in order to assess the validity of the proposed predictive control scheme. Each group of simulations enables a particular feature to be underlined.

- ✓ Figures 4-6 show the role of the precision parameter ε. For, several values of $\varepsilon = 0.05$ and 0.15 are used.
- ✓ Figures 7-8 show the influence of the prediction horizon length.
- ✓ Finally, figure 9 shows the case where $X = 0.5$. Note how the control keeps the value of the voltage in the prescribed precision interval.

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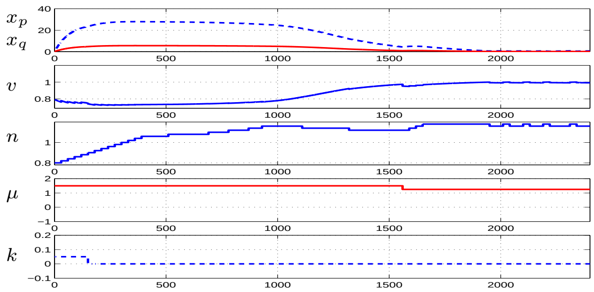

Figure 4: $X = 0.48$, $n(0) = 0.8$, $N_p = 20$ *and* $\varepsilon = 0.05$

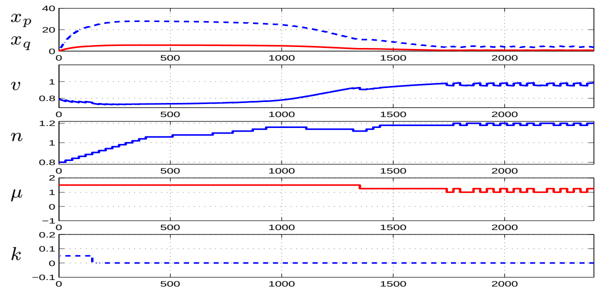

Figure 5: $X = 0.48$, $n(0) = 0.8$, $N_p = 20$ *and* $\varepsilon = 0.1$

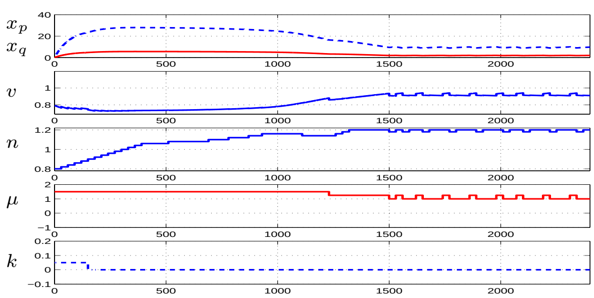

Figure 6: $X = 0.48$, $n(0) = 0.8$, $N_p = 20$ *and* $\varepsilon = 0.15$

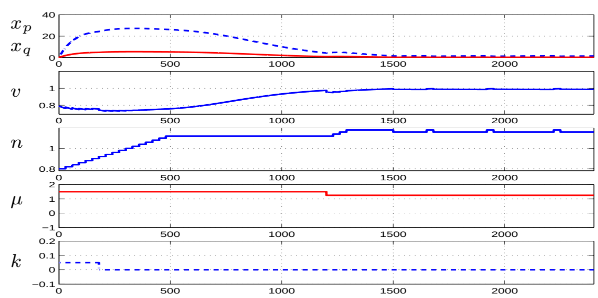

Figure 7: $X = 0.48$, $n(0) = 0.8$, $N_p = 40$ *and* $\varepsilon = 0.05$

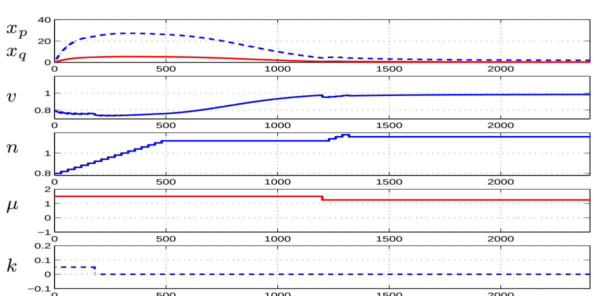

Figure 8: $X = 0.48$, $n(0) = 0.8$, $N_p = 80$ *and* $\varepsilon = 0.05$

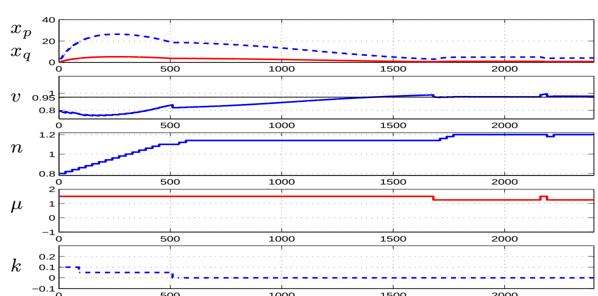

Figure 9: $X = 0.5$, $n(0) = 0.8$, $N_p = 80$ *and* $\varepsilon = 0.05$