On friction compensation without friction model

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Abstract

This paper presents a new methodology for friction compensation that is not based on any friction model. This is done by using finite-terms Fourier series to approximate the friction term. Updating laws for the coefficients of the series are easily derived from a Lyapunov approach to guarantee asymptotic convergence of the tracking error. Computer simulations are then proposed in which friction term is generated by existing models (unknown to the controller) to illustrate the efficiency of the proposed approach in compensating friction effects without friction model. Extension to the case of unknown inertia is also proposed. Robustness against velocity measurement errors is investigated by simulation. Moreover, since no friction model is used in the proposed compensation scheme, the resulting controller can be used without any preliminary identification phase necessary in most of the existing approaches.

1 Introduction

Friction is inevitable in mechanical systems. It is a highly nonlinear phenomenon that causes deterioration in the performance of servomechanisms giving rise to control problems such as static errors, limit cycles [2] and stick-slip. Nowadays control applications requiring high precision at low velocity tasks cannot put up with such imperfections and compensation for these effects have to be performed.

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Passive techniques such as lubrication are only partially effective due to the sensitivity to wear and environmental changes [16; 12]. High bandwidth linear controllers are inadequate since they are over-sensitive to high frequency model uncertainties and sensor noises.

Based on the above facts, intensive researches have been oriented towards a deeper comprehension of this complex and unpredictable [4] phenomenon in order to derive the most realistic and faithful model that can be used by nonlinear controllers (see [20; 11; 3; 6] and more recently [5; 9; 18; 8; 21; 15]). Controllers based on such models have proved to be successful in highly increasing the precision and the robustness of the resulting system performances.

It is clear that an even short survey of model-based friction compensation controllers is beyond the scope of this paper that proposes a completely new free-model approach. Interested readers may consult the excellent surveys [3; 16] and more recently the detailed discussion presented in [21] where the most complete faithful model seems to be proposed.

The work presented in this paper is based on the following observations :

- The friction phenomenon being highly complex and nonlinear, it seems that faithful models need to be so. Furthermore, once a so complicated model have been obtained, its parameters need to be estimated. Unfortunately, as it has been rightly pointed out in [9], friction may change as a function of normal forces in contact, temperature variations, position, etc.

  Variations in any of these factors may change simultaneously the model parameters in a very complicated way. Corresponding convergent updating laws cannot be obtained without a-priori somehow authoritarian assumptions on the structure of parameters variation making updated parameters appearing in an affine way [9].

- In all existing models and even physically, the friction term, when regarded as a function of time appears to be a bounded piece-wise continuous function that satisfies the Dirichlet's conditions. Therefore, it can be transformed into the Fourier Series within any given time interval [13].

> This suggests the use of already known schemes that control nonlinear systems containing time-varying uncertainties as those proposed in [14]. Indeed, feedback laws based on the above schemes present an impressive capacity of matching and predicting unknown signals in relation with stabilization-like objectives.
>
> This paper shows that the friction compensation problem falls exactly in that class of problems for which the use of Fourier series is powerful. This results in a direct estimation of the friction term without any model. Indeed, such scheme can be qualified as a direct estimation scheme in comparison with existing schemes that estimate the friction terms by identifying the parameters of some a-priori structurally fixed friction model.

The advantages in using such direct free-model schemes is evident since the results is independent of friction models that can never be perfectly faithful. On the other hand, whatever is the model used in the control scheme, there is certainly some combinations of facts that make it inappropriate at least temporarily to correctly represent the friction term. Finally, with direct estimation, there is no need to exhibit assumptions on the structure of parameter variations to insure the convergence of the updating laws. Finally, since no a-priori friction model is used, no preliminary identifications are needed. This results in a great simplification of the overall compensation scheme making it easy to adopt and to use by practitioners.

It is worth noting that it is not in our intention to under-estimate the relevance of a deeper comprehension and a more perfect modeling of the friction phenomenon for simulation [7], prediction, dimensioning and many other sorts of engineering and scientific purposes. In particular, simulation-based validation of free-model control schemes (as it is the case in this paper) need to use faithful friction models to be convincing. We believe however that whenever only high performance closed loop tracking and regulation are concerned, friction modeling can be avoided.

The present paper is organized as follows. Section 2 states the problem under consideration. Some facts concerning Fourier series expansions are recalled in section 3. In Section 4, a solution of the friction compensation problem with perfectly known inertia is presented together with simulations comparing the performance of the proposed approach to the widely accepted model-based adaptive approach proposed in [9]. The case of unknown inertia is handled in section 5. In section 6, simulations are proposed to validate the results of section 5 and to demonstrate the robustness of the proposed compensation scheme against velocity measurement errors. Finally conclusions are drawn in section 7.

2 Problem statement

the dynamics of the process in which the friction is present is assumed to be represented by

$$m\ddot{x} = -F + u \tag{1}$$

where m is the mass, F the friction force, u is the actuator force and x is the position to be controlled. It goes without saying that (1) can be directly used in the case of angular control with m replaced by the moment of inertia J and u representing the torque.

The control objective is to track a given reference signal x_r in the following two cases :

- STANDARD PROBLEM: The mass m is perfectly known.
- PROBLEM WITH UNKNOWN MASS: Only a lower bound $\underline{m}$ on the mass is given, that is $m \geq \underline{m}$.

Solutions for the two above problems are presented in sections 4 and 5 and the resulting two controllers are denoted Controller#1 and Controller#2 respectively. These solutions resort to Fourier series expansions, that is why the following section recalls some facts that are related to this technique.

3 Function approximation with Fourier series

Any piecewise continuous real-valued function $f(t)$ satisfying the Dirichlet's conditions may be represented within any finite time-interval of length T by a Fourier series of the form [13] :

$$f(t) = a_0 + \sum_{i=1}^{\infty}\Big(a_i \cos\omega_i t + b_i \sin\omega_i t\Big) \tag{2}$$

where $\omega_i := 2i\pi/T$ ($i \in \mathbb{N}$). The values a_i, b_i are called the Fourier coefficients. Now, putting :

$$
\begin{aligned}
Z_F(t) &:= \Big[1, \cos\omega_1 t, \sin\omega_1 t, \dots, \cos\omega_{n_F} t, \sin\omega_{n_F} t\Big]^T \in \mathbb{R}^{2n_F+1} \qquad (3)\\
W_f &:= \Big[a_0, a_1, b_1, \dots, a_{n_F}, b_{n_F}\Big] \in \mathbb{R}^{2n_F+1} \qquad (4)
\end{aligned}
$$

equation (2) can be rewritten in the following form

$$f(t) = W_f^T Z_F(t) + \epsilon(t) \qquad (5)$$

where $\epsilon(t)$ is the error due to the truncation satisfying [19] :

$$|\epsilon| \le \sum_{i>n_F} (|a_i| + |b_i|)$$

and since $|a_i|$ and $|b_i|$ are directly linked to the signal's energy at the corresponding frequency $2i\pi/T$, taking n_F sufficiently large ensures small approximation error ϵ since any physical signal is necessary practically band limited.

4 Solution for the standard problem (Controller#1)

Recall that the standard problem amounts to find a control law that makes the position x of the dynamical system (1) tracking some reference signal x_r assuming that the mass m is perfectly known and that both x and $v = \dot{x}$ are perfectly measured.

Let us denote the tracking error by e, namely

$$e := x - x_r$$

consider the stabilizing surface :

$$S = \dot{e} + \lambda_e e \quad ; \quad \lambda_e > 0$$

computing the time-derivative of S gives

$$\dot{S} = -\frac{F}{m} + \frac{1}{m}u - \ddot{x}_r + \lambda_e(v - \dot{x}_r) \qquad (6)$$

therefore, if one has a good estimation $\hat{F}_1$ of

$$F_1 := \frac{F}{m}$$

a suitable stabilizing feedback may be given by :

$$u = m\Big[\hat{F}_1 + \ddot{x}_r - \lambda_e(v - \dot{x}_r) - \lambda_s S\Big] \tag{7}$$

since (7) imposes the following dynamics on S :

$$\dot{S} = \hat{F}_1 - F_1 - \lambda_s S \tag{8}$$

Now, suppose that n_F is chosen sufficiently large for the following to hold with good precision :

$$F_1(t) = W_{F_1}^T Z_F(t) \quad ; \quad W_{F_1} \in \mathbb{R}^{2n_F+1}(\text{fixed}) \tag{9}$$

using the following parameterization for $\hat{F}_1$:

$$\hat{F}_1 := W_{\hat{F}_1}^T(t) Z_F(t) \quad ; \quad W_{\hat{F}_1} \in \mathbb{R}^{2n_F+1} \tag{10}$$

equation (8) becomes :

$$\dot{S} = \Big[W_{\hat{F}_1} - W_{F_1}\Big]^T Z_F - \lambda_s S =: \tilde{W}_{F_1}^T Z_F - \lambda_s S \tag{11}$$

where $\tilde{W}_{F_1} := W_{\hat{F}_1} - W_{F_1}$. Now, let us consider the following nonnegative function :

$$V := \frac{1}{2}S^2 + \frac{1}{2}\tilde{W}_{F_1}^T Q_F \tilde{W}_{F_1} \tag{12}$$

and compute its time derivative under the control (7) in which (10) is injected :

$$\dot{V} = \tilde{W}_{F_1}^T \Big[S Z_F + Q_F \dot{\tilde{W}}_{F_1}\Big] - \lambda_s S^2 \tag{13}$$

and using the fact that $\dot{\tilde{W}}_{F_1} = \dot{W}_{\hat{F}_1}$, (13) becomes :

$$\dot{V} = \tilde{W}_{F_1}^T \Big[S Z_F + Q_F \dot{W}_{\hat{F}_1}\Big] - \lambda_s S^2 \tag{14}$$

This suggests the following updating law for $W_{\hat{F}_1}$:

$$\dot{W}_{\hat{F}_1} = -SQ_F^{-1} Z_F(t) \tag{15}$$

Indeed, with this updating law, one has :

$$\dot{V} = -\lambda_s S^2 \tag{16}$$

which implies by the invariance principle that $\lim_{t\to\infty} S = 0$ and hence $\lim_{t\to\infty} e = 0$ (see remark 1 hereafter).

To sum up, the solution of the standard friction compensation problem is given by the following dynamic output feedback :

$$u = m\Big[W_{\hat{F}_1}^T Z_F + \ddot{x}_r - \lambda_e(v - \dot{x}_r) - \lambda_s S\Big] \tag{17}$$

$$\dot{W}_{\hat{F}_1} = -SQ_F^{-1} Z_F(t) \tag{18}$$

$$S := (v - \dot{x}_r) + \lambda_e(x - x_r) \tag{19}$$

Remarque 1. *To be completely rigourous, it is worth noting that the last asymptotic argument used to conclude (*$\lim_{t\to\infty} S = 0$*) omits the fact that equation (16) is valid only over the finite time interval over which a Fourier series with constant coefficient vector* W_{F_1} *is used to approximate* F_1 *[see (9)] and therefore, asymptotic argumentation cannot rigourously hold in this context.*

Nevertheless, if T *and* n_F *are chosen sufficiently large, equation (16) ensures that* S *goes close to* 0 *inside the time interval under concern. At the beginning of the next time interval of length* T*, a new fixed* W_{F_1} "takes place" *and one may expect a slight perturbation to appear periodically each* T *seconds. Fortunately, since this perturbation appears in the neighborhood of* $S = 0$*, one may expect that it has minor influence on the tracking performance. Simulations given hereafter confirms this intuition.*

On the other hand, because of the truncation of higher frequency terms and many other factors, the updating law needs to be slightly modified using dead zone or σ*-modification [17] in practical implementation to avoid possible parameter drifting.*

4.1 Validation of the standard problem's solution (Controller#1)

In this section, simulations are proposed to validate the solution proposed in the preceding section concerning the standard problem. In these simulations, comparisons are done with the compensation scheme proposed in [9]. This enables an implicit comparison with the compensation strategy based on adaptive nonlinear control with persistent excitation proposed in [16] since in the later, simulations show that the proposed method is slightly better than the non adaptive version of [9] that was proposed in [10].

4.1.1 Description of the simulations protocol

Two low velocity reference trajectories $x_r(t)$ have been tested.

- In the first, the following sinusoidal signal is used

$$x_r(t) = 0.05\sin(2\pi t/10) \tag{20}$$

- In the second, the periodic signal proposed in [9] is used with a lower amplitude in order to obtain very low velocities :

$$x_r(t) = 0.05\sin(2\pi t/5)\sin(2\pi t/100) \tag{21}$$

The friction model's structure used in the simulation is the one used to design the compensation scheme in [9], namely :

$$\dot{z} = v - \frac{\sigma_0}{\alpha_0 + \alpha_1 e^{(-v/v_0)^2}} z|v| \tag{22}$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \alpha_2 v \tag{23}$$

In [9], two one-parameter based adaptive friction compensation schemes have been proposed to handle either variations in only the static parameters (σ_0,σ_1) or variations in all the friction parameters appearing in (22)-(23).

Since in the two proposed schemes, only one-parameter based adaptation is used, one may expect that when all the friction model's parameters (σ_0,σ_1, α_0, α_1, and α_2) indeed change, the adaptive compensation scheme of [9] performs less better that in the case where only σ_0 and σ_1 change while all the others are supposed to be perfectly known.

Based on the above discussion, the case where only static friction parameters σ_0 and σ_1 is chosen in the comparison in order to favour the model-based friction compensation scheme of [9]. Note that This scheme is already favoured by the use of the friction model's structure it uses in its own design. Indeed, it has been shown in [21; 1] that the friction model (22)-(23), while globally satisfactory, still presents some shortcomings preventing the accurate prediction of friction behaviour under some circumstances (over-dissipativity in presliding, fixed transition curve shape).

The nominal values for the friction model parameters have been taken equal to those used in [9], namely :

$$\alpha_0^{nom} = 0.285 \quad ; \quad \alpha_1^{nom} = 0.05 \quad ; \quad \alpha_2^{nom} = 0.01 \tag{24}$$

$$v_0^{nom} = 0.01 \quad ; \quad \sigma_0^{nom} = 260 \quad ; \quad \sigma_1^{nom} = 0.6 \tag{25}$$

while dependance w.r.t the velocity sign has been introduced according to the table I of [9].

The parameter values of the compensation scheme given in [9] have been used, namely

$$\omega_0 = 20 \quad ; \quad \xi = 0.999 \quad ; \quad n = 1.2 \quad ; \quad k = 1 \quad ; \quad \gamma = 10$$

Note that ω_0, ξ and n are used to tune the PID related terms in the compensation action (see [9] for a complete description of the corresponding compensation scheme).

The friction model has been slightly detuned by changing only σ_0 and σ_1 by +30% and +5% respectively. All the other friction model's parameters are supposed to be exactly known by the model-based compensation controller while the free-model compensation scheme proposed in this paper ignores naturally everything about the friction model used to generate the friction term in the simulations. The free-model compensation scheme used the following parameters for all the related experiments

$$T = 25 \quad ; \quad n_F = 10 \quad ; \quad Q_F = 0.001 \quad ; \quad \lambda_e = \lambda_s = 50 \tag{26}$$

4.1.2 Simulation Results and discussion

✠ tracking reference signal (20)

Figure 1 shows the tracking performances for the PID without compensation (a), the detuned model-based controller of [9] (b) and the free-model compensator (Controller#1) proposed in the preceding section (c). Friction force estimations of both the model-based controller and the free-model compensation scheme (Controller#1) are shown on Figure 2. Figure 3 shows a detailed view of the tracking error under the two different compensation schemes. Finally, Figure 4 shows evolution of some of the Fourier coefficients vector $W_{\hat{F}} = mW_{\hat{F}_1}$.

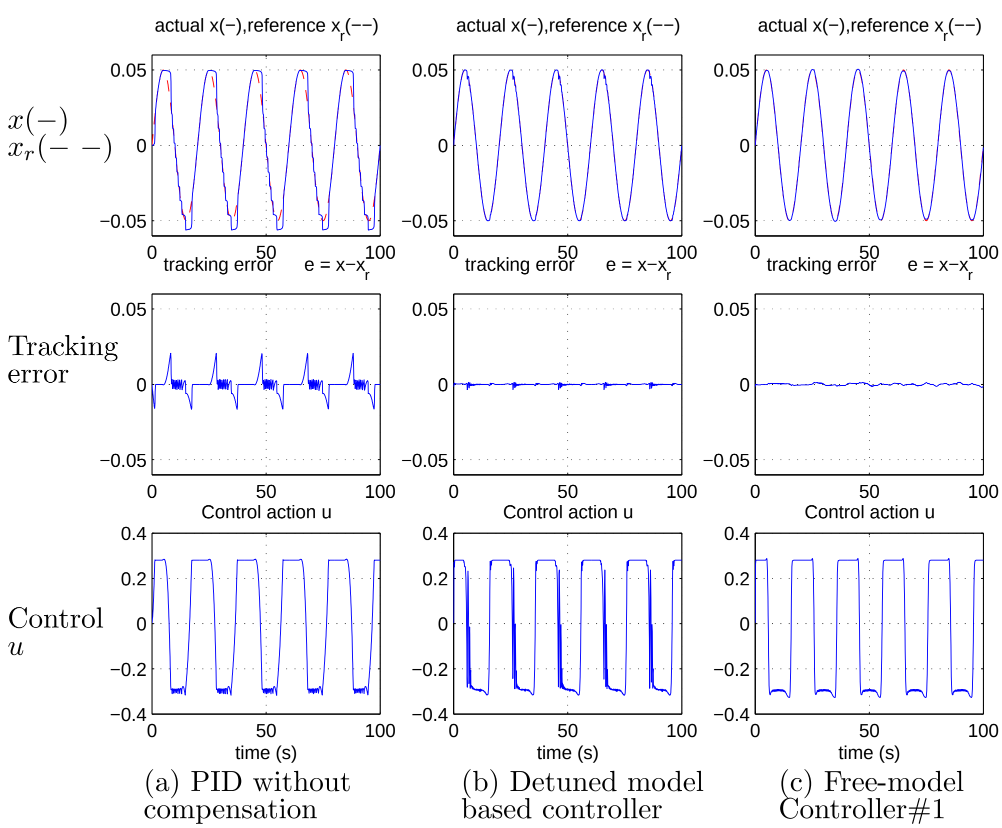


Fig. 1: Comparison of controllers performances for the reference signal (20)

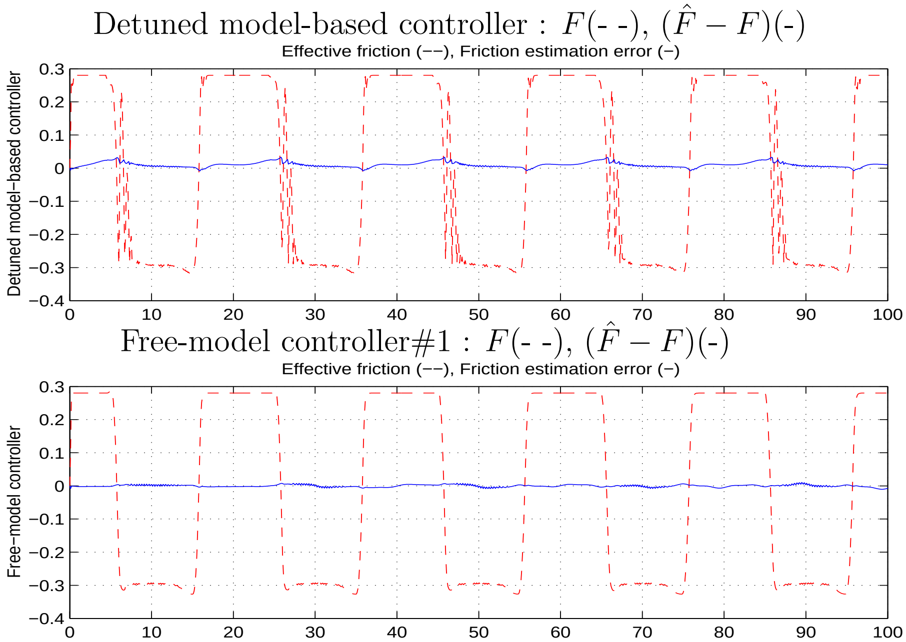


Fig. 2: Friction and friction estimation errors under the reference signal (20)

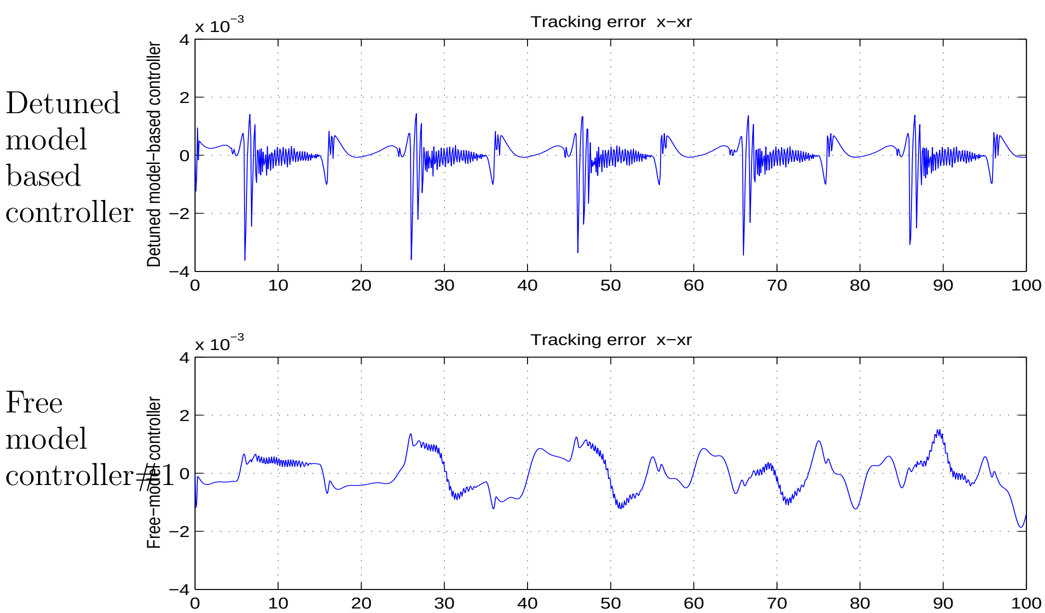


Fig. 3: Detailed view of tracking errors under the reference signal (20)

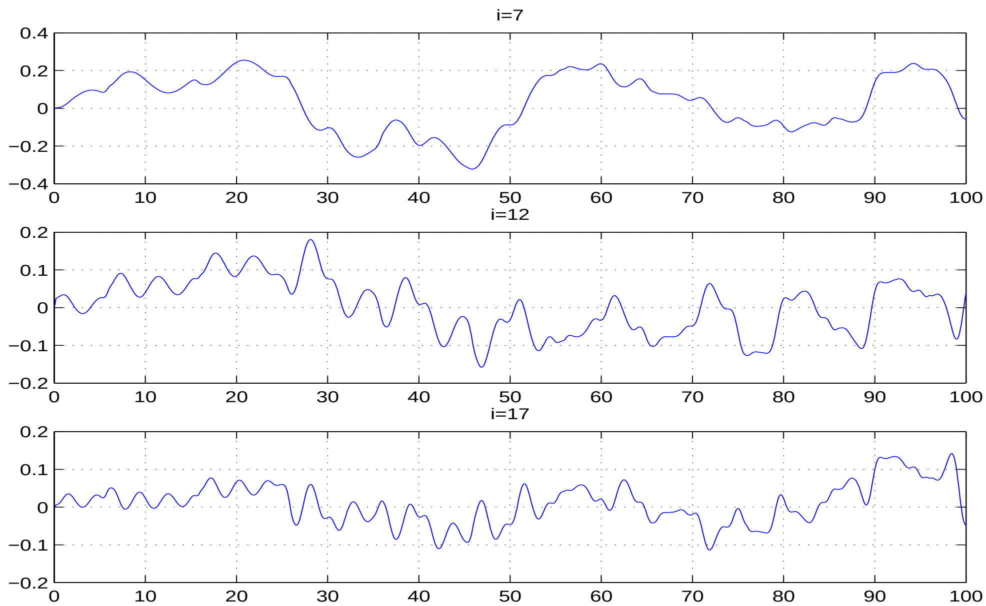
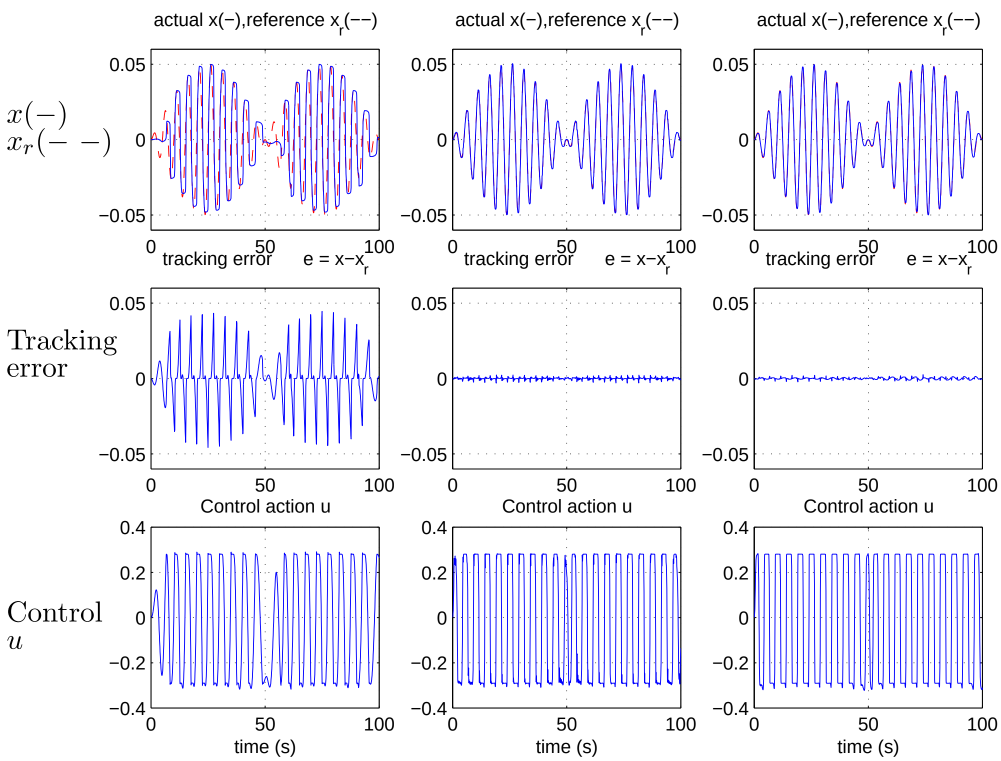


Fig. 4: Some of the components of the Fourier coefficient vector $W_{\hat{F}} = mW_{\hat{F}_1}$ under the reference signal (20)

✠ tracking reference signal (21)

Figure 5 shows the tracking performances for the PID without compensation (a), the detuned model-based controller of [9] (b) and the free-model compensator (Controller#1) proposed in the preceding section (c). Friction force estimations of both the model-based controller and the free-model compensation scheme (Controller#1) are shown on Figure 6. Figure 7 shows a detailed view of the tracking error under the two different compensation schemes. Finally, Figure 8 shows evolution of some of the Fourier coefficients vector $W_{\hat{F}} = mW_{\hat{F}_1}$.



(a) PID without compensation (b) Detuned model based controller (c) Free-model Controller#1

Fig. 5: Comparison of controllers performances for the reference signal (21)

As expected, when low velocities arise ($v \approx 0$), tracking error increases in the absence of friction compensation while it is maintained equally small [see Figures 3 and 7] by both nonlinear model-based and free-model controller.

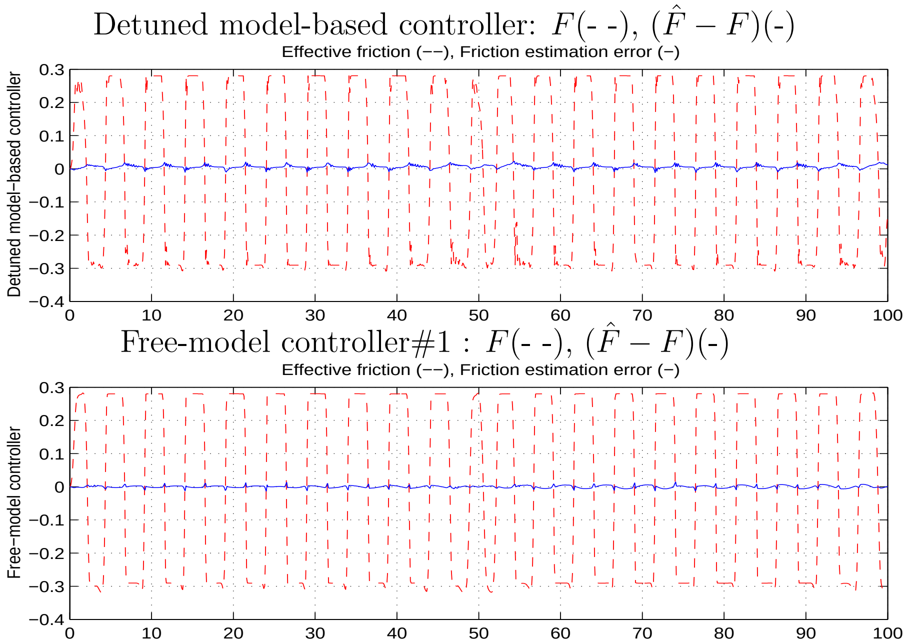


Fig. 6: Friction and friction estimation errors under the reference signal (21)

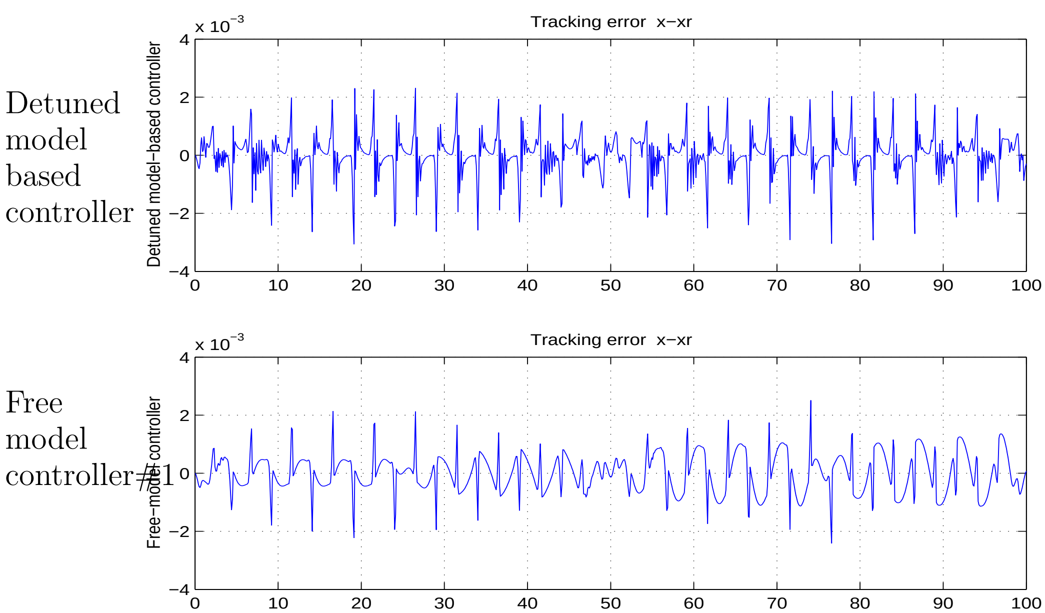


Fig. 7: Detailed view of tracking errors under the reference signal (21)

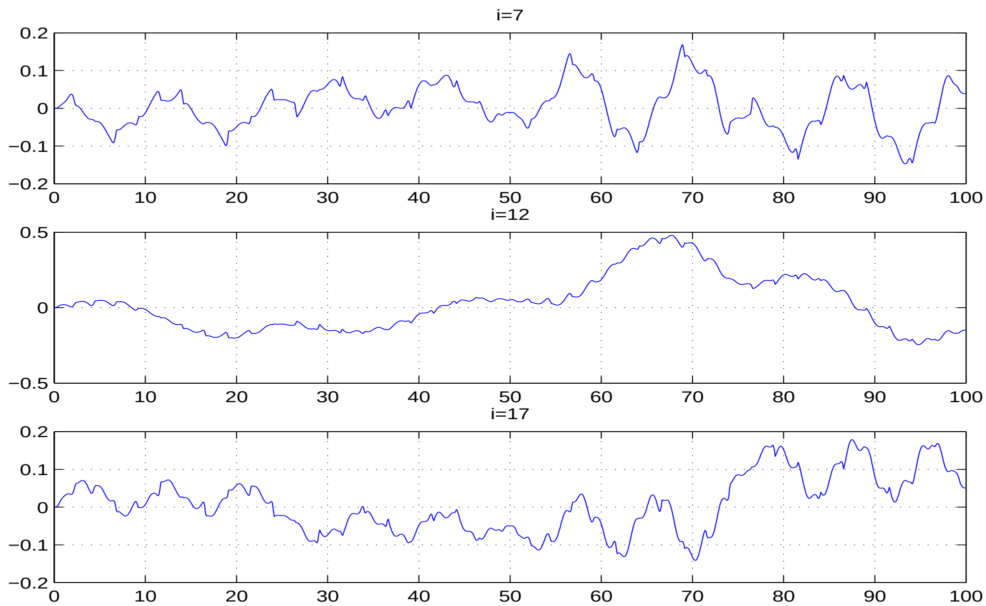


Fig. 8: Some of the components of the Fourier coefficient vector $W_{\hat{F}} = mW_{\hat{F}_1}$ under the reference signal (21)

Examination of Figures 2 and 6 shows that the free-model controller possesses an excellent friction estimation capacity since the friction estimation error is almost always in the neighborhood of 0. Furthermore, comparison between the dynamics of the friction estimation [see Figures 3 and 7] shows that the free-model estimation seems to be less "*nervous*" than the estimation associated to the detuned model-based scheme. The same can be said about the resulting control action.

Finally, a careful examination of the evolution of the Fourier coefficients depicted on Figures 4 and 8 shows a change of behaviour at instant $t_k = kT = 25k$ as expected following the discussion of remark 1. Note however that this T-periodic sudden changes have no major effects on the overall tracking quality (see Figures 3 and 7)

5 Solution for the friction compensation problem with unknown inertia (Controller#2)

In this section, the mass (or the moment of inertia for rotating systems) is supposed to be unknown. Only a lower bound $\underline{m}$ of m is given such that :

$$m \geq \underline{m} \tag{27}$$

Using the notations of section 4, equation (6) is still valid :

$$\dot{S} = \frac{1}{m}\Big[-F + u - m\big(\ddot{x}_r - \lambda_e(v - \dot{x}_r)\big)\Big] \tag{28}$$

In order to simplify the expressions, the following notation is used for the measured quantity $\ddot{x}_r - \lambda_e(v - \dot{x}_r)$:

$$E = \ddot{x}_r - \lambda_e(v - \dot{x}_r) \tag{29}$$

so that (28) becomes

$$\dot{S} = \frac{1}{m}\Big[-F + u - mE\Big] \tag{30}$$

giving rise to the following control strategy

$$u = \hat{F} + \hat{m}E - \hat{m}\lambda_s S \tag{31}$$

where $\hat{m}$ and $\hat{F}$ are some instantaneous estimations of m and F respectively. Using (31) in (30) gives

$$\dot{S} = \frac{1}{m}\Big[\hat{F} - F + (\hat{m} - m)E - \hat{m}\lambda_s S\Big] \tag{32}$$

therefore,

$$\dot{S} = \frac{1}{m}\Big[\tilde{W}_F^T Z_F + \tilde{W}_m E - \hat{m}\lambda_s S\Big] \tag{33}$$

where, following the terminology of section 4,

$$\hat{F} = W_{\hat{F}}^T Z_F \quad ; \quad \tilde{W}_F := W_{\hat{F}} - W_F \quad ; \quad \tilde{W}_m := \hat{m} - m$$

Now consider the nonnegative function V given by :

$$V = \frac{1}{2}S^2 + \frac{1}{2m}\left[\tilde{W}_F Q_F \tilde{W}_F + q_m \tilde{W}_m^2\right] \tag{34}$$

where $Q_F \in \mathbb{R}^{(2n_F+1)\times(2n_F+1)}$ and $q_m \in \mathbb{R}$ are positive definite. Computing the time derivative of V gives :

$$\dot{V} = \frac{1}{m}\left[\tilde{W}_F^T\Big(Z_F S + Q_F \dot{W}_{\hat{F}}\Big) + \tilde{W}_m\Big(SE + q_m\dot{\hat{m}}\Big) - \hat{m}\lambda_s S^2\right] \tag{35}$$

which suggests the following adaptation laws for $W_{\hat{F}}$ and $\hat{m}$

$$\dot{W}_{\hat{F}} = -\Big(Q_F^{-1} Z_F\Big)S \tag{36}$$

$$\dot{\hat{m}} = \begin{cases} -SE/q_m & \text{if } \hat{m} > \underline{m} \\ 0 & \text{if } \hat{m} = \underline{m} \text{ and } SE < 0 \\ -SE/q_m & \text{if } \hat{m} = \underline{m} \text{ and } SE \geq 0 \end{cases} \tag{37}$$

Indeed, (37) ensures that $\hat{m}$ remains greater than the lower bound $\underline{m}$ (provided that initial value of the estimation $\hat{m}$ meets this requirement) while examination of (36)-(37) and (35) shows that $\dot{V}$ satisfies the following inequality

$$\dot{V} = \begin{cases} -\frac{\hat{m}}{m}\lambda_s S^2 & \text{if } \hat{m} > \underline{m} \\ -\frac{\hat{m}}{m}\lambda_s S^2 + \tilde{W}_m SE & \text{if } \hat{m} = \underline{m} \text{ and } SE < 0 \\ -\frac{\hat{m}}{m}\lambda_s S^2 & \text{if } \hat{m} = \underline{m} \text{ and } SE \geq 0 \end{cases}$$

and using the fact that $\tilde{W}_m = \hat{m} - \underline{m} \geq 0$, it comes that in all cases one has

$$\dot{V} \leq -\frac{\hat{m}}{m}\lambda_s S^2 \leq -\frac{\underline{m}}{m}\lambda_s S^2 \tag{38}$$

and asymptotic convergence holds.

To sum up, the solution of the friction compensation problem with unknown inertia is given by the following dynamic output feedback :

$$u = W_{\hat{F}}^T Z_F + \hat{m}[\ddot{x}_r - \lambda_e(v - \dot{x}_r)] - \hat{m}\lambda_s S \tag{39}$$

$$\dot{W}_{\hat{F}} = -\left(Q_F^{-1} Z_F\right) S \tag{40}$$

$$\dot{\hat{m}} = \begin{cases} -SE/q_m & \text{if} \quad \hat{m} > \underline{m} \\ 0 & \text{if} \quad \hat{m} = \underline{m} \text{ and } SE < 0 \\ -SE/q_m & \text{if} \quad \hat{m} = \underline{m} \text{ and } SE \geq 0 \end{cases} \tag{41}$$

$$E = \ddot{x}_r - \lambda_e(v - \dot{x}_r) \tag{42}$$

$$S := (v - \dot{x}_r) + \lambda_e(x - x_r) \tag{43}$$

6 Validation of the free-model controller under unknown inertia

In this section, simulations are proposed to first illustrate the efficiency of the solution proposed in section 5 in handling uncertainties on the system's inertia and then to investigate the robustness of the controllers proposed in both sections 4 and 5 against velocity measurement errors. This is done while comparing the performances of these two controllers to that of the detuned model-based controller that has been used in section 4.

For easy references, the controllers proposed in sections 4 and 5 are denoted by Controller#1 and Controller#2 respectively. Reference signal (21) is used in all the simulations.

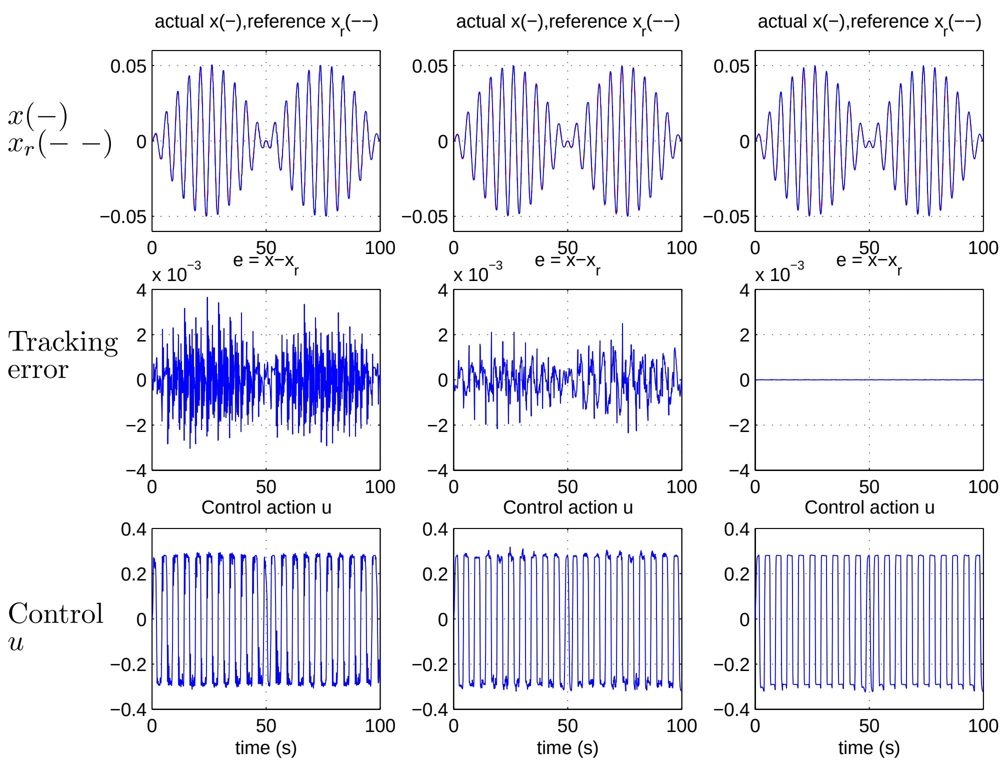
- ✓ Simulations with erroneous mass and perfect velocity measurements [Figures 9 and 10]

  In this simulation, the effective mass m used in the simulation is 4 times greater than the nominal mass m_{nom} used in all the controllers (as a constant value in the detuned model-based controller and controller#1 and as an initial value of $\hat{m}$ in controller#2)

$$m = 4 \times m_{nom} = 4 \times 0.0022$$

Figure 9 shows the performances of the three controllers in tracking the reference signal (21). The order of magnitude of the tracking error corresponding to the Controller#2 is ($\sim 10^{-6}$) while that of the detuned model-based controller and Controller#1 is ($\sim 10^{-3}$).

Figure 10 shows the evolution of the estimated mass $\hat{m}$ used by Con-



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 9: Comparison of controllers performances for the reference signal (21) under perfect velocity measurements with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$)

troller#2. Note that the result of section 5 do not guarantee the convergence of $\hat{m}$ to m but only the convergence of the tracking error [see equation (38)].

This simulation suggests that the mass adaptation mechanism included in free-model Controller#2 enables a noticeable improvement of the tracking performances.

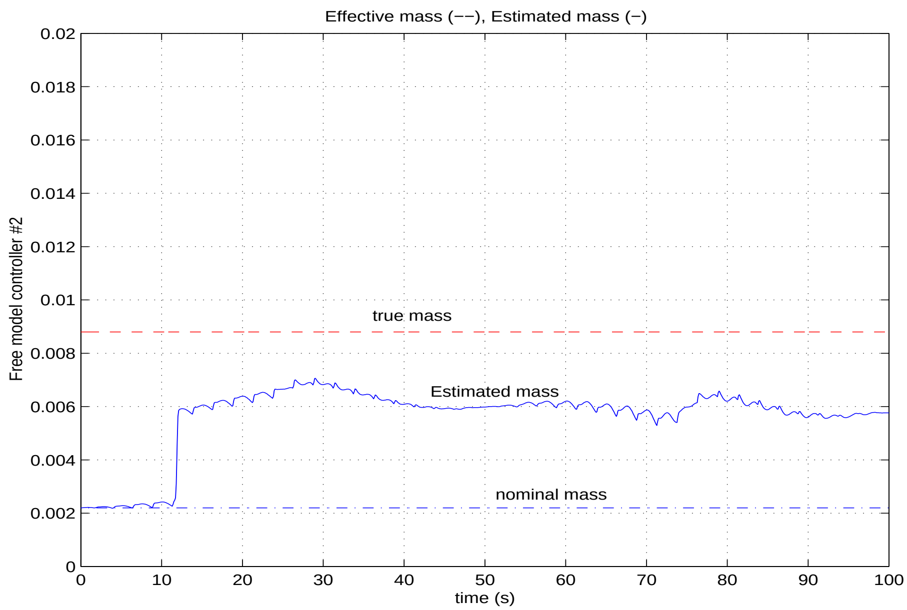


Fig. 10: Evolution of the estimated mass $\hat{m}$ used by Controller#2 when tracking the reference signal (21) using perfect velocity measurements and erroneous initial estimated mass $\hat{m}(0) = 0.25m$

✓ Simulations with erroneous mass and unperfect velocity measurements : Case of absolute error [Figures 11-14]

In this simulation, an absolute measurement error is introduced such that :

$$v_m = v - \varepsilon_v \quad ; \quad \varepsilon_v = 0.005 \tag{44}$$

where $v = \dot{x}$ is the true velocity while v_m is the measured velocity used by the controllers. This simulates a constant sensor offset. The error on the mass is maintained as in the preceding simulation. Figure 11 shows the performances of the three controllers in tracking the reference signal (21) while Figure (12) shows a detailed view of the tracking error profiles.

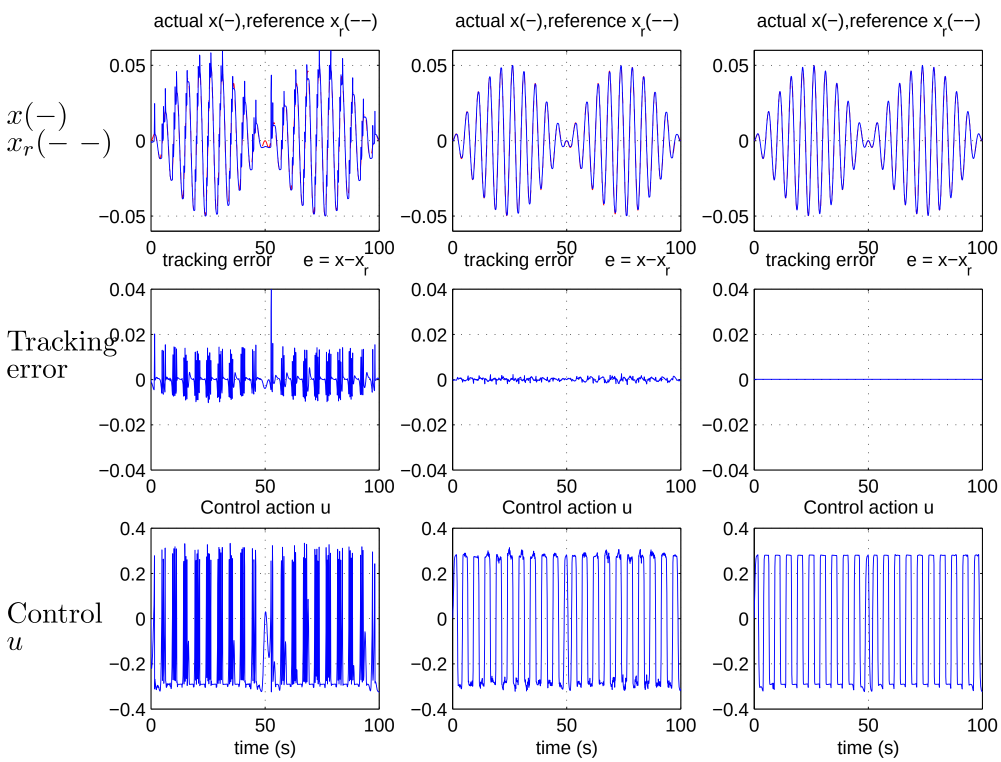
This simulation suggests that free-model controllers #1 and #2 are more robust to offset-like velocity measurements errors that the model-based controller. Errors on the friction force prediction obtained by the different controllers are shown on Figure 13. The very good ability of the free-model controllers to compensate the velocity measurement errors comes from the fact that the effects of these errors are interpreted as a bad estimation of the friction term or the value of the system's inertia and corresponding compensation is performed since adaptation laws are computed based on the tracking criterion. Figure 14 shows the behaviour of the estimated mass $\hat{m}$ used by Controller#2 during this experiment.

✓ Simulations with erroneous mass and unperfect velocity measurements : Case of relative error [Figures 15-16]

In this last experiment, a relative error is used on velocity measurements, namely

$$v_m = (1 + \varepsilon_v)v \quad ; \quad \varepsilon_v = 0.15 \tag{45}$$

Results are presented on Figure 15 and a detailed view of tracking error is given on Figure 16.



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 11: Comparison of controllers performances for the reference signal (21) with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$) and the constant offset error (44) on velocity measurements

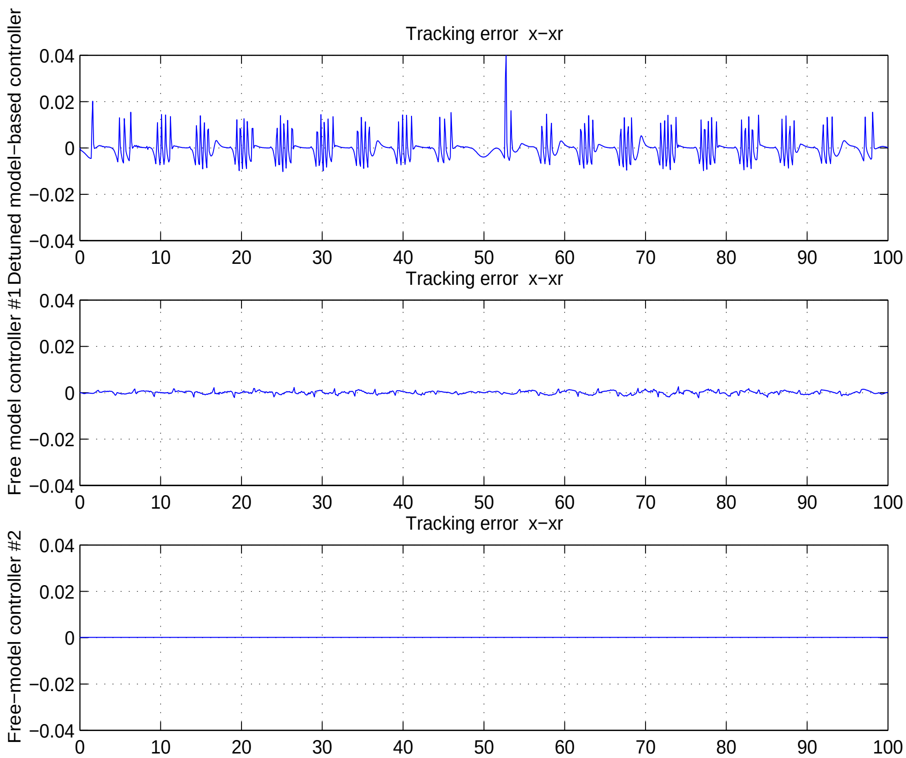


Fig. 12: Comparison of controllers performances for the reference signal (21) with erroneous mass m_{nom} used by the controllers (true mass $m = 4 \times m_{nom}$) and the constant offset (44) on the velocity measure

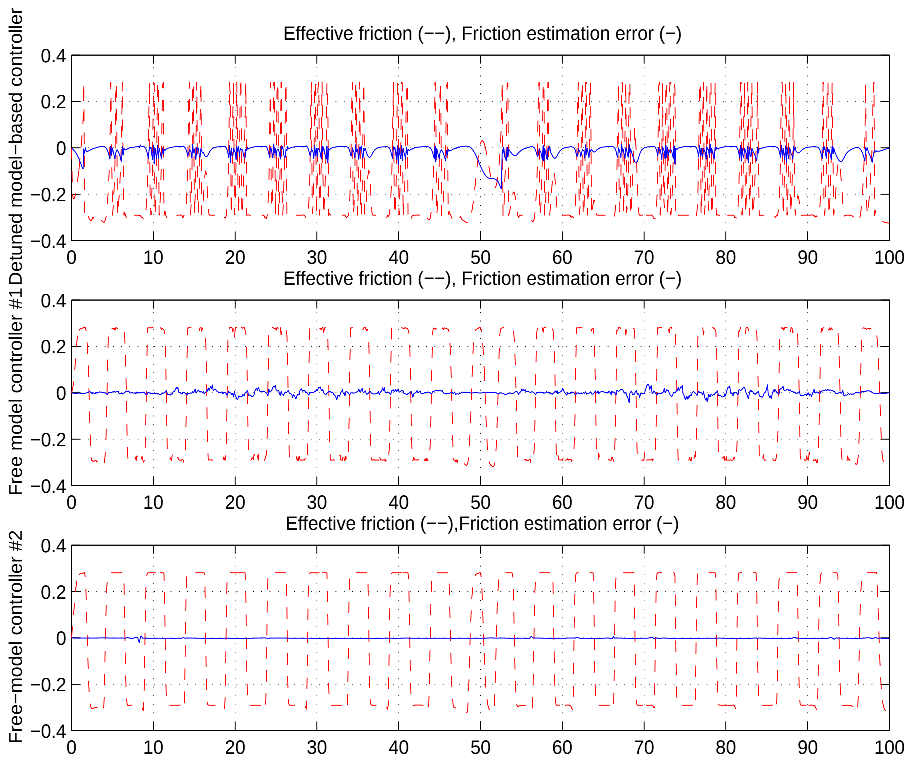


Fig. 13: Comparison of friction force prediction for the reference signal (21) with erroneous mass m_{nom} (true mass $m = 4 \times m_{nom}$) and the constant offset error (44) on the velocity measure

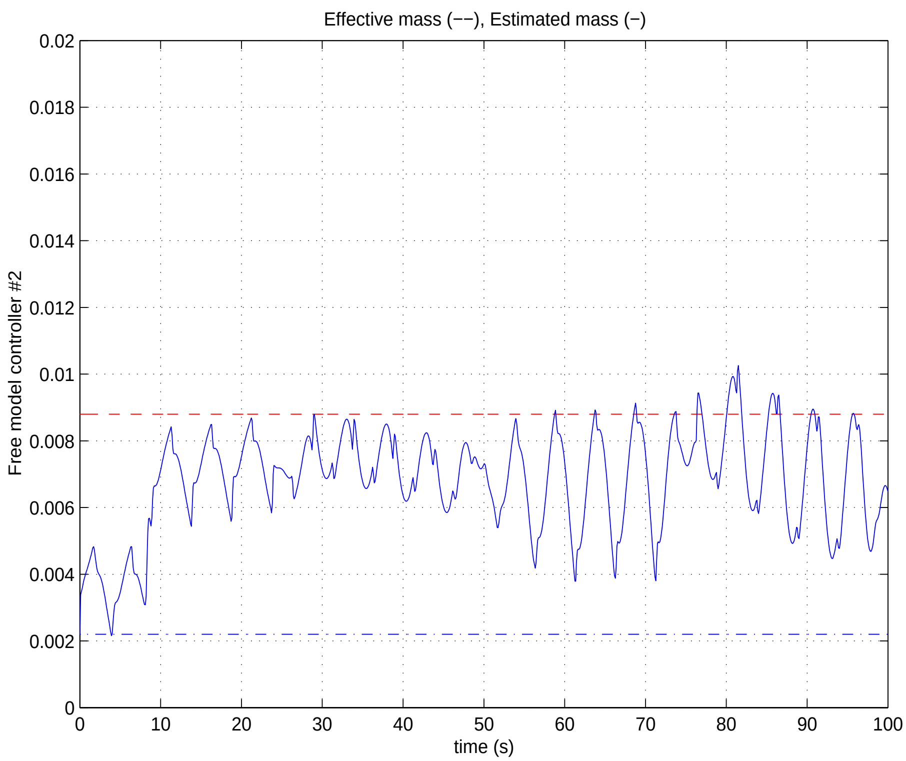
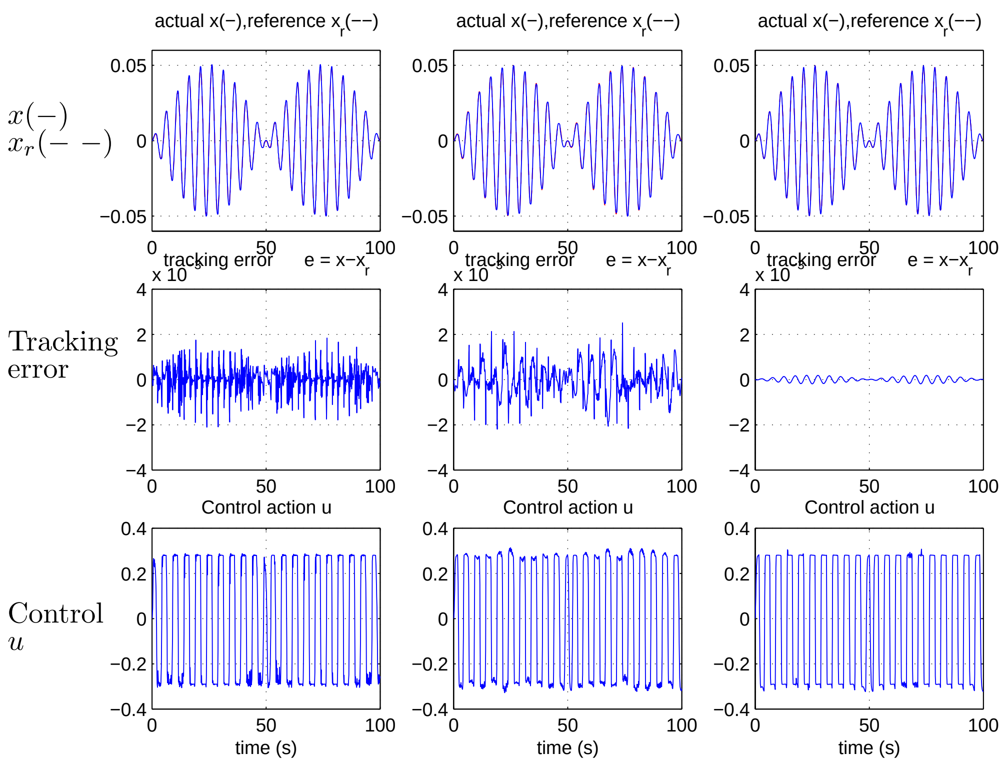


Fig. 14: Evolution of the estimated mass $\hat{m}$ used by Controller#2 when tracking the reference signal (21) with erroneous mass m_{nom} (true mass $m = 4 \times m_{nom}$) and the constant offset error (44) on the velocity measure



(a) Detuned model based Controller (b) Controller#1 (c) Controller#2

Fig. 15: Comparison of controllers performances for the reference signal (21) with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$) and relative measurement error given by (45) on velocity measurements

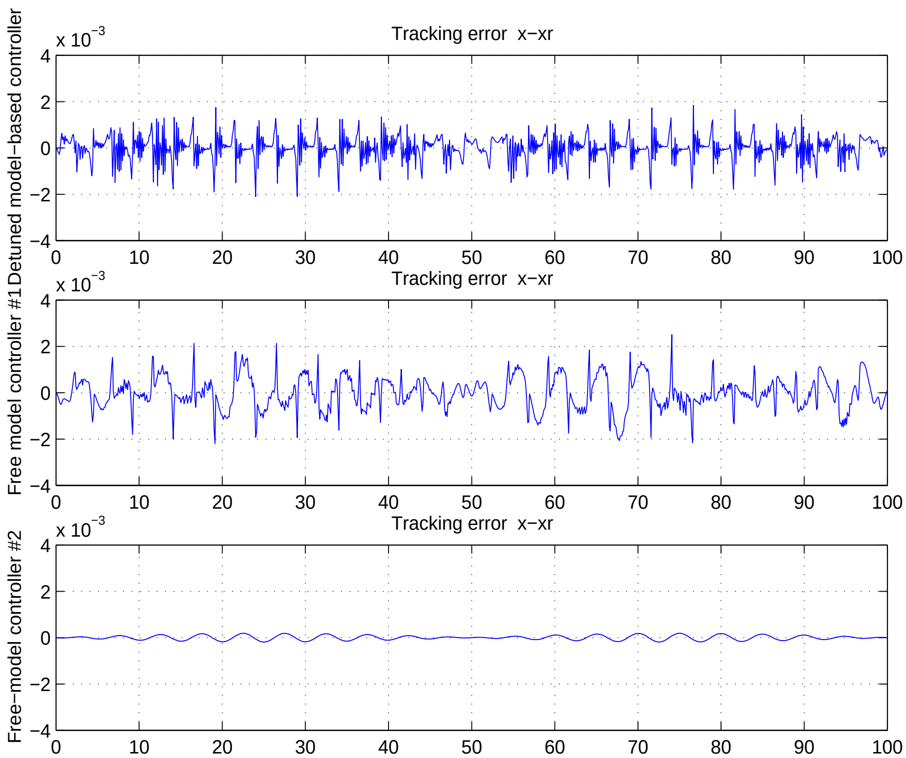


Fig. 16: Comparison of controllers performances for the reference signal (21) with erroneous nominal mass m_{nom} (true mass $m = 4 \times m_{nom}$) and relative measurement error (45) on the velocity measure

7 Conclusion

In this paper, a free-model friction compensation scheme is proposed based on Fourier series expansion of the unknown friction term. Updating laws for the Fourier series coefficients are obtained by Lyapunov approach. This yields a dynamic output feedback. Two different controllers have been obtained depending on the exact or uncertain knowledge of the system's inertia.

Tracking performances of the free-model controllers so-obtained have been compared to an existing and widely appreciated model-based nonlinear adaptive controller [9]. The friction model used in the simulation to generate the friction term (unknown by the free-model controllers) is the one used in [9] to design the controller. However the static friction parameters σ_0 and σ_1 have been detuned by 30% and 5% respectively to simulate real world uncertainties and identification errors.

Since the model-based controller used in the comparison has been designed to compensate for uncertainties in only the static parameters and since the structure of the friction model used to generate the friction term is exactly the same than the one used in the model-based controller design, it may be argued that the proposed comparisons are quite favourable to the model-based controller. Even so, comparison shows that the free-model controllers performs at least equally well in the absence of mass uncertainty and using perfect velocity measures and much better when uncertainties on system's inertia and/or errors on velocity measurement are introduced.

To sum up, model-free compensation schemes presented in this paper seems to be very promising from both performance, robustness and implementation point of view since no friction model is needed nor preliminary identification experiments are necessary.

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