On Higher Order Dynamics in the Fundamental Equation of Frequency in Islanded Microgrids

J. Dobrowolski * M. Alamir ** S. Bacha *** D. Gualino * M-X. Wang *

* Schneider Electric Industrie. 37 Quai Paul Louis Merlin. 38000 Grenoble. (jean.dobrowolski(david.gualino(miao-xin.wang))@schneider-electric.com)
** CNRS, University of Grenoble Alpes. 11 rue des Mathmatiques, 38000 Grenoble, France (mazen.alamir@gipsa-lab.grenoble-inp.fr)
*** Laboratoire de Genie Electric de Grenoble. 21 avenue des Martyrs, 38000 Grenoble, France (Seddik.Bacha@g2elab.grenoble)

Abstract: In islanded micro grids, the stability of the frequency has to be enhanced by the distributed producers. Moreover, distributed control schemes are highly appealing for evident robustness and flexibility reasons. This is generally achieved by using the fundamental frequency model in which the derivative of the grid frequency is proportional to the power unbalance in the grid. This paper discusses the relevance of this simple model when high gain closed-loop system is designed. In particular, the presence of higher order terms in the frequency dynamics that should be carefully taken into account in the distributed frequency control design is proved and appropriate frequency control law is proposed and validated.

Keywords: Microgrid, Decentralized control, Identification, Grid Frequency, DER, Control under Communication Constraints.

1. INTRODUCTION

Since the 2000s, the increasing awareness of environmental issues, combined with economic factors and the proliferation of distributed renewable resources enhanced an increasing interest in distributed grids (Zeh et al., 2016; Asmus, 2010; Hartono et al., 2013; Hatziargyriou et al., 2009; Bacha et al., 2015).

Microgrids are localized and autonomous electrical distribution networks composed of Distributed Energy Resources (DER) including storage devices and loads. They can be connected to a larger power system or isolated voluntarily, to supply grid-off areas or islands (Jing et al., 2016), or involuntarily because of larger power blackout or disturbance (Hatziargyriou et al., 2007).

Islanded microgrids are composed of renewable energy producers which have to ensure grid stability, prompting a lot of research on frequency control (Tang et al., 2016; Reil, 2016; Simpson-Porco et al., 2015). Among others, the control based on the sole frequency measurement is an appealing concept to allow Plug & Play controlled architectures (Dörfler et al., 2014).

Due to the complexity of frequency dynamics of a whole microgrid and the targeted Plug & Play nature of their connection, the most common way to control frequency is to describe its dynamics as proportional to the unbalance between the total power $P_p$ provided by the producers and the power $P_l$ consumed by the loads (Delille et al., 2012; Zhao et al., 2016), namely:

$$\dot{f} = \alpha[P_p - P_l]$$ (1)

where $\alpha$ is a parameter that depends on the grid inertia. This equation comes from the approximation done when translating the swing equation from one generator to a multi-machine system leading to an equivalent generating unit describing the behavior of $n$ generators (Anderson and Fouad, 2008; Rudyte and Mihalic, 2011).

However, our recent investigation on real-life microgrid frequency control testbed suggested that an aggressive control (high closed-loop bandwidth) can destabilize the grid frequency which is not in accordance with what is expected, should the simplified frequency equation (1) be correct. This observation incited us to further investigate the structure of the frequency dynamics and the appropriate control gain design which is the main topics of the present contribution.

More precisely, we consider the model of a simple microgrid composed of two Gensets build up used the widely used PLECS toolbox of MATLAB/SIMULINK (Section 2). Based on this model and a concrete PLL-based grid frequency measurement, an identified dynamic model is derived for the variable $e = f - \alpha(P_p - P_l)$ (Section 3). This variable obviously represents the dynamic of the error involved when using the simplified frequency equation (1). The results clearly show that a higher order model is necessary when high frequency dynamics are included in the scenarios. In section 4, the resulting model is analyzed and the bandwidth limitation induced by the higher dynamics components of the new frequency model is discussed based
on the recent results of (Alamir et al., 2017). It is in particular shown that necessary conditions can be derived for the parameter of frequency-measurement-based control law to enhance stability. Finally, Section 5 concludes the paper and gives the roadmap for further investigation.

2. SIMULATION MODEL

To identify the grid frequency dynamics, we need a simulator of a multisources microgrid. We choose to represent a microgrid composed of two gensets connected to a load. To analyze the behavior depending on the DERs size, we use one genset of 45kVA (MeccAlte, 2013) and one of 100kVA (Olympian, 2014) for a total microgrid power capacity of 145 kVA.

A genset is an autonomous system which is able to produce electricity, by transforming mechanical energy of a thermal engine into electrical energy via a synchronous machine. To simplify the complex modeling of synchronous machines (Zhong and Weiss, 2009), we choose to use PLECS-MATLAB toolbox which proposes a modeling block of synchronous machines whose behavior is explained in (PLECS, 2016).

The engine rotates at a speed \( \omega \) which evolves proportionally to the unbalance between the mechanical torque \( \Gamma_m \) applied by a motor and the electromagnetic torque \( \Gamma_e \) required by load via the synchronous machine. Namely

\[
J \frac{d\omega}{dt} = \Gamma_m - \Gamma_e - D\omega
\]

where \( J \) is the inertia and \( D \) is the damping coefficient. Sum of producers mechanical torque without damping \( \Gamma_m - D\omega \) is the mechanical image of \( P_p \) and sum of electromagnetic torque \( \Gamma_e \) is the mechanical image of \( P_l \). The engine rotational speed \( \omega \) is associated to the genset frequency \( f = (N\omega)/(2\pi) \) with \( N \) the number of poles of the synchronous machine.

To ensure a balanced sharing of the load power among DERs, and to control frequency in case of an islanding microgrid, the well known primary droop control is used (Simpson-Porco et al., 2015; Zhong, 2013). Primary control is based on the active/reactive power decoupling (Zhong, 2013) and calculates a rotational speed reference \( \omega_{ref} \) in order to keep \((P_m, \omega)\) on the droop line defined by

\[
\omega = \omega_0 - m \times P
\]

where \( \omega_0 \) is the nominal grid frequency and \( m \) is the droop slope. The rationale behind the droop characteristic is that an increase of the measured power \( P_m \) is the sign of an increase of the load power \( P_l \) which should induce a drop in the frequency (following the basic presumed dynamics (1)) in order to increase and share the total produced power \( P_p \) between each producers.

Note that in the presence of two gensets, the direct link between the grid frequency \( f \) and the rotational speed \( \omega \) is no more a priori valid since now we have two \( \omega \)'s that cannot be instantaneously equal. Instead, the grid frequency is evaluated using a Phase Lock Loop (PLL) designed as explained in (Hadjidemetriou et al., 2013). This commonly admitted technique is used in the simulator to deliver the grid frequency for the forthcoming identification step. This results in the simulation framework depicted in figure 3.

3. DYNAMIC FREQUENCY MODEL

Now that we defined a microgrid simulator and an identification scenario, our aim is to investigate to which extent does the simplified dynamics (1) represent the measured frequency in the simple two gensets micro grid described in the preceding section.

Figure 4 checks the validity of the basic equation (1) by comparing \( \dot{f} \) to \( \alpha (P_p - P_l) \) where \( \alpha \) is computed to give a better possible fit. The latter is defined as follows:

\[
\text{fit} = \left[ 1 - \sum_{t=1}^{t_{max}} \frac{|\dot{f}(t) - \alpha (P_p(t) - P_l(t))|}{|\dot{f}(t)| + \varepsilon} \right] \times 100
\]

From Figure 4, it can be clearly observed that the basic equation (1) is not sufficient to capture the transient in the measured frequency and that a higher order model seems to be necessary.

Considering the residual

\[
e = \dot{f} - \alpha (P_p - P_l)
\]

of the satisfaction of the basic frequency equation (1) as the output of a dynamical system to be identified and the derivative of the total produced power and load power, namely \( \dot{P}_p \) and \( \dot{P}_l \) as input signals, it is possible to use a state-space identification subroutines such as the Mat-
Figure 3. Schematic view of the simulation used to identify the two-genets grid frequency dynamics.

Checking the equation $\dot{f} = \alpha (P_p - P_l)$: Comparison between $\dot{f}$ and the best fit of $\alpha (P_p - P_l)$.

Lab/N4SID in order to derive an identified input/output system of the following form:

$$\dot{\eta} = A\eta + Bu$$ \quad \text{where} \quad u := \begin{bmatrix} \dot{P}_p \\ \dot{P}_l \end{bmatrix} \quad (6)$$

$$e = C\eta + Du \quad (7)$$

Combining (5) with (6)-(7) in which the input/output gain matrix $D = 0$ (by the identification results), the complete identified dynamics of the grid frequency can be given by:

$$\dot{\eta} = A\eta + B_1\dot{P}_p + B_2\dot{P}_l$$

$$f = \alpha (P_p - P_l) + C\eta \quad (9)$$

Examination of the eigenvalues of the state matrix $A$ involved in the resulting model (8) shows that this matrix is Hurwitz with oscillating modes. Figure 6 shows the impulse response of (7). The real system input is the load power $P_l$ so there is no sense to apply impulse directly on $(\dot{P}_p - \dot{P}_l)$ because $P_p$ and $P_l$ are linked by mechanical equation. It comes clearly that the settling time of the second term in (9) which represents the error of the commonly used simple dynamics (1) has an order of magnitude of 1.5 sec.

This explains why an agressive closed-loop control that ignores the presence of this extra dynamics can destabilize the behavior of the frequency. This destabilizing effect is accentuated by the fact that the derivative of the produced power $\dot{P}_p$ (that is viewed as the control variable in the frequency stabilizing feedback) is precisely the leading term in (8) that excites the internal state $\eta$ of the frequency dynamics once the variation of the load power stops.
With the same droop slope $P(8)-(9)$, the total power that producers have to provide in order to regulate the frequency of the extended model (8)-(9) making uncontrollable the dynamics of the system (8)-(9) using the same law (11) in which the high gain $\omega_{ref}$ is shifted by $\delta \omega_0$ (see 7) which becomes the so-called secondary control variable that manipulates the power produced by each individual Genset.

Figure 6. Impulse response of (8) for an impulse on the load power $P_l$.

Recall that we are interested in output feedback control that regulates the grid frequency around its reference value $f_0 \Leftrightarrow \omega_0$ (typically 50Hz). To do so, we follow the suggestion by (Simpson-Porco et al., 2015) by adding a secondary control $\delta \omega_0$ that modifies the offset $\omega_0$ of the droop characteristic of equation (3). By doing so, the reference value $\omega_{ref}$ is shifted by $\delta \omega_0$ (see 7) which becomes the so-called secondary control variable that manipulates the power produced by each individual Genset.

In order to regulate the frequency of the extended model (8)-(9), the total power that producers have to provide $P_p$ has to be estimated. This global estimation allows to share $P_p$ between producers and to compute the secondary control variable $\delta \omega_0$. Considering that all producers have the same droop slope $m$, we share $P_p$ equitably relative to producers size by applying the following secondary control:

$$\delta \omega_0 = m \times P_p \times \frac{P_i}{\mathcal{P}}$$  \hspace{2cm} (10)

with $\mathcal{P}$ the total power of all producers, $P_i$ the size of $i$th producer and $\delta \omega_0$ its secondary control variable.

Assuming that $P_p$ is bounded by the grid capacity $[P_{min}, P_{max}]$, we use the control law proposed in (Alamir, 2015) and (Alamir et al., 2017), namely:

$$P_p = S(\lambda(f_0 - f) + z)$$  \hspace{2cm} (11a)

$$\dot{z} = \lambda_f (P_p - z)$$  \hspace{2cm} (11b)

With $\lambda$ and $\lambda_f$ two design parameters, $z$ the internal state of the controller while $S$ the saturation function according to $S(v) =$:

$$S(v) := \min\{P_{max}, \max\{P_{min}, v\}\}$$  \hspace{2cm} (12)

Note that the dynamics is steered only by the measured signal $f$ and the internal state $z$. Therefore, they can be run by all the gensets so that they can have a common estimation $z$ of the load power.

Note that in (Alamir, 2015), it has been shown that if (1) were true, there would be no theoretical limitation on $\lambda$ and therefore one can control the frequency as tightly as desired. On the other hand, (Alamir et al., 2017) investigated the case where internal dynamics is present and derived appropriate bounds on $\lambda$ and $\lambda_f$ that leads to a controlled closed-loop performance. These two situations are analyzed in the next sections for the micro grid context while (Alamir, 2015) and (Alamir et al., 2017) give general purpose results.

4.1 Without limitation on control gains

The design proposed in (Alamir, 2015) holds for (1) provided that $\alpha \in [\alpha, \bar{\alpha}], \eta \in [P_{min}, P_{max}]$ such that:

$$P_{max} - P_{max} \geq \varphi_+ > 0$$  \hspace{2cm} (13)

$$P_{min} - P_{min} \geq \varphi_- > 0$$  \hspace{2cm} (14)

These assumptions hold for the micro grid case with $P_p \in [0, 145]$ kW, $P_i \in [3.5, 121.8]$ kW and $\alpha \in [0.1]$. Under these conditions, (Alamir, 2015) states that if (1) holds then for any $\lambda > 0$, if the following conditions hold:

$$\lambda_f < \left[ \min \left( \frac{\min\{\varphi_+, \varphi_-\} \alpha}{P_{max} - P_{min}}, \frac{\alpha}{4} \right) \right] \times \lambda$$  \hspace{2cm} (15)

$$\left| \frac{dP_i}{dt} \right| \leq \sigma P_i$$  \hspace{2cm} (16)

then the dynamic feedback law defined by (11) leads to a tracking error $e_f = f - f_0$ such that:

$$\lim_{t \to \infty} \left| e_f \right| \leq \frac{\sigma P_i}{\lambda \times \lambda_f}$$  \hspace{2cm} (17)

For the scenario of Figure 2, one has $\sigma P_i \leq 510500$ and $\min \left( \frac{\min\{\varphi_+, \varphi_-\} \alpha}{P_{max} - P_{min}}, \frac{\alpha}{4} \right) = 0.0241$. Therefore, to ensure an asymptotic tracking error lower than 0.1 Hz, the conditions on $\lambda$ are:

$$\frac{\sigma P_i}{\lambda \times \lambda_f} \leq 0.1 \Leftrightarrow \lambda \geq \sqrt{\frac{510500}{0.0241 \times 0.1}} = 14554$$  \hspace{2cm} (18)

Figure 8 shows a simulation of the closed-loop system (1) under (11) with $\lambda = 14600$ and $\lambda_f = 14600 \times 0.0240$. It can be observed that the predicted asymptotic bound on the tracking error holds.

Unfortunately, the simulation of the identified high order system (8)-(9) using the same law (11) in which the same ($\lambda, \lambda_f$) are used shows very bad behavior as it can be observed in Figure 9. This comes from the fact that the high gain $\lambda = 14554$ induces high oscillations on $\eta$ [through (8)] which is then injected in the frequency equation (9) making uncontrollable the dynamics of the frequency. Fortunately, the framework of (Alamir et al., 2017) precisely tackles this issue as it is shown in the next section.

4.2 With limitation on control gains

This section heavily involves the recent results of (Alamir et al., 2017) whose full statement lies beyond the scope
considering value, we consider a design variable \( \alpha \)
Therefore performance of control will be strongly linked to
of the present contribution. The general equation of the
with \( \lambda = 14600 \) and \( \lambda_f = 350.4 \)
of the present theory is:
Therefore performance of control will be strongly linked to
the lowest bound of \( \alpha \) the denominator of \( C_\eta \)
To fix this value, we consider a design variable \( \gamma \) in the system:
Stability of (19) is satisfied if that of (20) is satisfied considering \( \gamma \) as a time scale variable.
Only the main guidelines of the result are developed.
Note first of all that one obviously has:

\[
\begin{align*}
\frac{C_\eta(t)}{\alpha\gamma} &\leq c_0 + c_1 ||\dot{u}\gamma||_{\infty} \quad (21) \\
\frac{C_\eta(t)}{\alpha\gamma} &\leq d_0 + d_1 ||\dot{u}\gamma||_{\infty} \quad (22) \\
\end{align*}
\]

where \( c_0, c_1, d_0 \) and \( d_1 \) are given by:

\[
\begin{align*}
c_0 & := \epsilon + \sup_{t \geq 0} \frac{C}{\alpha\gamma} \int_0^t \exp^{\frac{A(t-\sigma)}{\gamma}} B_2 \hat{P}_1 d\sigma \quad (23a) \\
c_1 & := \sup_{t \geq 0} \frac{C}{\alpha\gamma} \int_0^t \exp^{\frac{A(t-\sigma)}{\gamma}} B_1 d\sigma \quad (23b) \\
d_0 & := \epsilon + \sup_{t \geq 0} \frac{C}{\alpha\gamma} \int_0^t \exp^{\frac{A(t-\sigma)}{\gamma}} B_2 \hat{P}_1 d\sigma \quad (23c) \\
d_1 & := \sup_{t \geq 0} \frac{C}{\alpha\gamma} \int_0^t \exp^{\frac{A(t-\sigma)}{\gamma}} B_1 d\sigma \quad (23d) \\
\end{align*}
\]

Under (21)-(22), together with the bounds on \( \alpha, P_l \) and
\( P_p, \) one can rephrase the main result of (Alamir et al., 2017) by stating that if the following conditions hold:

1. \( c_0 < \min\left\{ \frac{\beta^-}{\gamma}, \frac{\beta^+}{\gamma} \right\} \)
2. \( \lambda < \lambda^* \) where \( \lambda^* \) is given by:

\[
\lambda^* = \sup \left\{ \sigma < \frac{1}{\pi c_1} \inf_{\lambda \in (0, \sigma)} \phi(\lambda) \leq 0 \right\} \quad (24)
\]

in which \( \phi \) is defined according to:

\[
\phi(\lambda) = \left[ \frac{\varrho}{\gamma} - c_0 - \lambda \left( \frac{\beta^+ + c_0}{\gamma} - \frac{c_1 \pi}{1 - \lambda \pi c_1} \right) \right] \alpha \quad (25)
\]

where \( \Delta_P = P_{\text{max}} - P_{\text{min}}, \varrho = \min\{\varrho^-, \varrho^+\} \) and \( \beta = P_{\text{max}} - P_{\text{min}} + \max\{\varrho^-, \varrho^+\} \).
3. \( \lambda_f < \lambda \times \phi(\lambda) \)

(4) The dynamic of \( P_l \) respect to \( \frac{dP_l}{dt} \leq \frac{\sigma_{P_l}}{\gamma} \)

Then according to (Alamir et al., 2017) the stability of
(20) is satisfied and so the tracking error \( e_f \) of system
(19) with control (11) satisfies:

\[
\lim_{t \to \infty} |e_f| \leq \frac{\sigma_{P_l} + d_0 + d_1 \sigma_u}{\lambda \times \lambda_f} \quad (26)
\]

where \( \sigma_u \) is given by:

\[
\sigma_u = \frac{\lambda \pi \varrho + c_0 + \lambda_f \Delta_a}{1 - \lambda \pi c_1} \quad (27)
\]

Using the identified system (8) and (9) \( c_0 = 2.456 \times 10^{-2}, \)
\( c_1 = 1.156 \times 10^{-4}, d_0 = 3.8754 \times 10^{-2} \) and \( d_1 = 2.435 \times 10^{-4} \) with \( \gamma = 70. \) The definition of \( \lambda^* \) is obtained through
the evolution of the function \( \phi(\lambda) \) depicted in Figure 10. Indeed, this figure clearly shows that \( \lambda^* = 1400 \)
(maximum value of \( \lambda \) such that \( \phi(\lambda) \geq 0 \)). Using \( \lambda = 0.8\lambda^* = 1120 \) leads to \( \lambda_f = \phi(\lambda) = 34. \) Injecting these values in (26)-(27) gives the guaranteed asymptotic bound
\( \lim_{t \to \infty} |e_f| \leq 1.3 \) which is confirmed by the simulation of Figure 11. Note that this bound holds even if \( P_f \) keep
varying continuously. If \( P_f \) becomes constant, the bound is given by

\[
\lim_{t \to \infty} |e_f| \leq 0.13
\]

Note that as many theoretical bounds, the asymptotic bound predicted by the theory is an upper pessimistic
bound that is based on many worst-case inequalities used
in the derivation of the theoretical statement. It gives however a safety bound that the practitioner can use with
confidence in the choice of the control parameters design.

5. CONCLUSION AND FUTURE WORK

In this paper, the problem of distributed control of
frequency in micro grids is investigated. In particular, it has
been shown that the fundamental equation of frequency in
micro grids is only valid for a limited bandwidth beyond
which high order dynamics have to be taken into account in
the control design. Moreover, an appropriate recently
developed framework (Alamir et al., 2017) has been adapted
to the micro grid context leading to a simple dynamic
Future investigation concerns the application of the main result to a real-life tested that is currently under construction at Schneider Electric company.

REFERENCES


